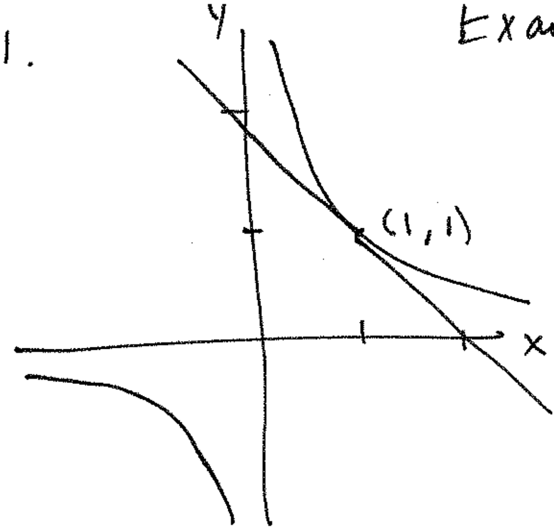
Exam 1 Solutions (141 F12)

1.



Try different values of x:

$x = 2$: $y = \frac{1}{2}$.

Secant line slope is $\dfrac{\frac{1}{2} - 1}{2 - 1} = \dfrac{\frac{1}{2}}{1} = \frac{1}{2}$

$x = \cancel{1000}\ \frac{11}{10}$, $y = \frac{10}{11}$:

Secant line slope is $\dfrac{\frac{10}{11} - 1}{\frac{11}{10} - 1} = \dfrac{\frac{-1}{11}}{\frac{1}{10}} = \dfrac{-10}{11}$

$x = \frac{101}{100}$, $y = \frac{100}{101}$.

Secant line slope is $\dfrac{\frac{100}{101} - 1}{\frac{101}{100} - 1} = \dfrac{\frac{-1}{101}}{\frac{1}{100}} = -\dfrac{101}{100}$

$x = \frac{99}{100}$, $y = \frac{100}{99}$

Secant line slope is $\dfrac{\frac{100}{99} - 1}{\frac{99}{100} - 1} = \dfrac{\frac{1}{99}}{\frac{-1}{100}} = \dfrac{-100}{99}$

It looks like the slopes are approaching -1.

So our tangent line is $(y - 1) = -1(x - 1)$

$y - 1 = -x + 1$

$y = -x + 2$.

2. $\lim_{x\to 1} \dfrac{x^3-1}{x^2-1} = \lim_{x\to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x\to 1} \dfrac{x^2+x+1}{x+1}$

$= \dfrac{1+1+1}{1+1} = \dfrac{3}{2}.$

3. $\lim_{x\to\infty} \sqrt{9x^2+x} - 3x = \lim_{x\to\infty} \dfrac{(\sqrt{9x^2+x}-3x)(\sqrt{9x^2+x}+3x)}{\sqrt{9x^2+x}+3x}$

$= \lim_{x\to\infty} \dfrac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x}$

$= \lim_{x\to\infty} \dfrac{x}{x\left(\sqrt{9+\frac{1}{x}}+3\right)}$

(Here, note that

$\sqrt{9x^2+x} = \sqrt{x^2}\sqrt{9+\frac{1}{x}}$

$= x\sqrt{9+\frac{1}{x}}$

because $x>0$.)

$= \lim_{x\to\infty} \dfrac{1}{\sqrt{9+\frac{1}{x}}+3} = \dfrac{1}{\sqrt{9+0}+3} = \dfrac{1}{6}.$

4. $f'(x) = \lim_{h\to 0} \dfrac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h}$

$= \lim_{h\to 0} \dfrac{(\sqrt{3(x+h)+1}-\sqrt{3x+1})(\sqrt{3(x+h)+1}+\sqrt{3x+1})}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}$

$= \lim_{h\to 0} \dfrac{(3(x+h)+1)-(3x+1)}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}$

$= \lim_{h\to 0} \dfrac{3x+3h+1-3x-1}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})} = \lim_{h\to 0} \dfrac{3h}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}$

$= \lim_{h\to 0} \dfrac{3}{\sqrt{3(x+h)+1}+\sqrt{3x+1}} = \dfrac{3}{2\cdot\sqrt{3x+1}}.$

5. y is a constant. The graph is flat.

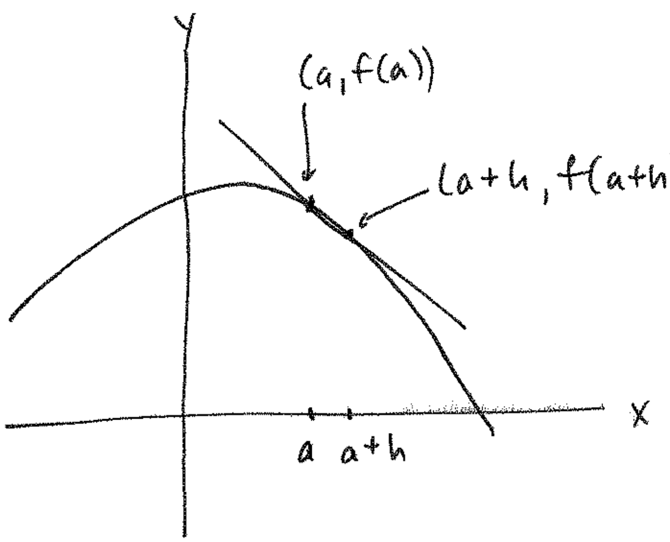
$$\frac{dy}{dx} = 0$$

6. $$\frac{dy}{dx} = \frac{(1-x^2)\frac{d}{dx}(x^3) - x^3\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)(3x^2) - x^3 \cdot (-2x)}{(1-x^2)^2}$$

$$= \frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{3x^2 - x^4}{(1-x^2)^2}.$$

7. By definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.



Draw the secant line between $(a, f(a))$ and $(a+h, f(a+h))$. Its slope is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h}.$$

As we take the point $a+h$ closer and closer to a, the secant line approaches the tangent line, so $\frac{f(a+h) - f(a)}{h}$ approaches $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$.

8. When a function is flat its derivative is zero
when it is increasing its derivative is positive
when it is decreasing its derivative is negative.

Where the function a reaches its minimum, neither b nor c is zero, so neither is the derivative of f.

So $a = f''$.

When a is negative, b is decreasing (and c is not).

So $b' = a$ and $b = f'$.

By elimination $c = f$. Note that the height of b matches the slope of c.