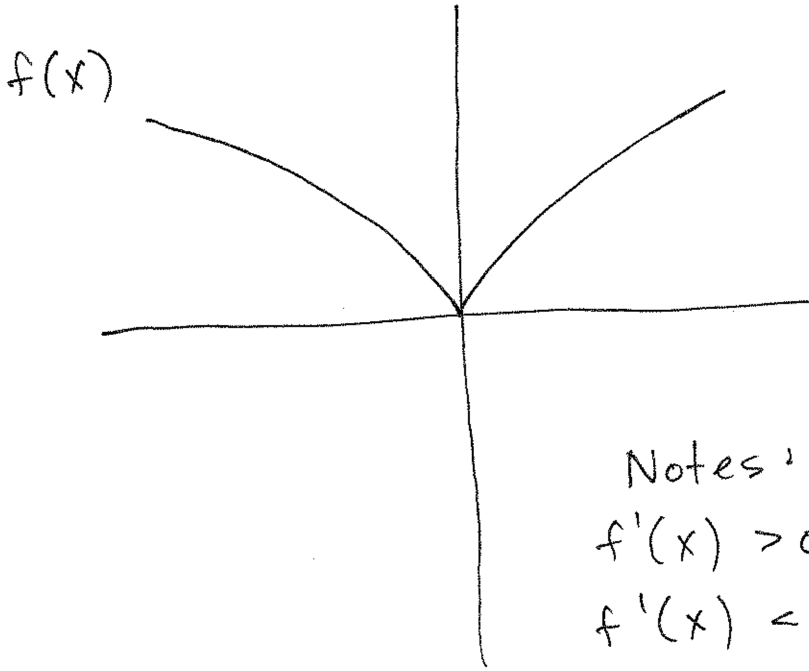
1.



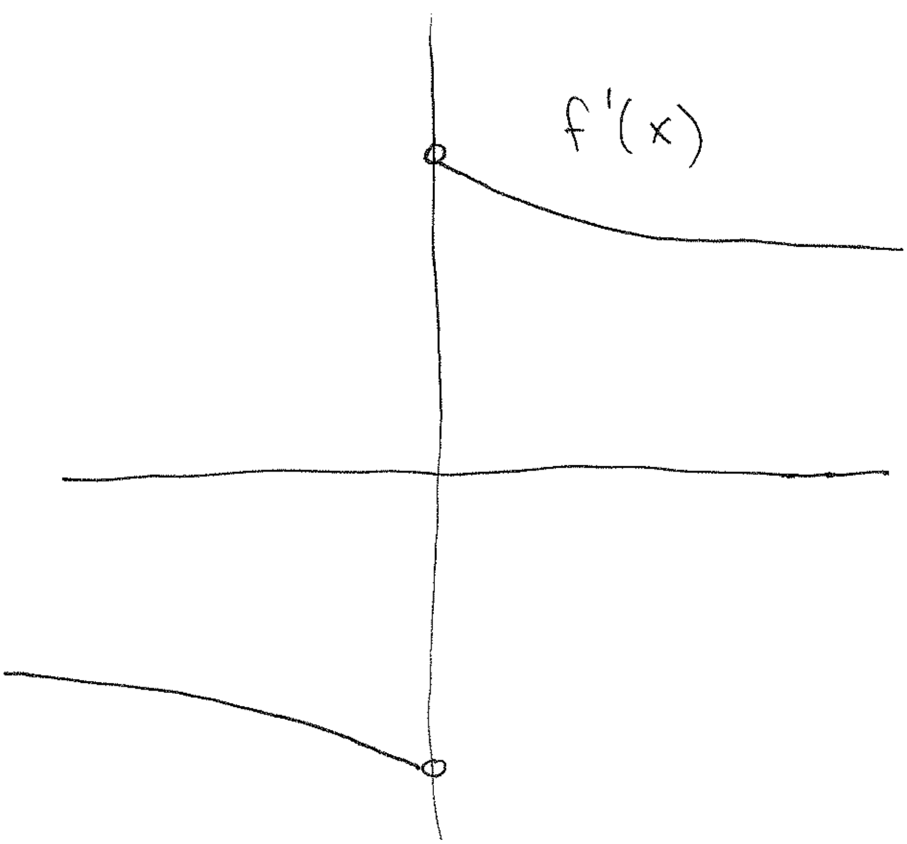
Notes:

$f'(x) > 0$ for $x > 0$

$f'(x) < 0$ for $x < 0$

$|f'(x)|$ is largest for x close to 0, but does not decay fast as $x \to \infty$ or $x \to -\infty$

$f'(0)$ is not defined because there is a "cusp".



Note: It is not clear from the picture of $f(x)$ whether there is a vertical asymptote at $x = 0$.

2. $$f'(x) = \lim_{h \to 0} \frac{(x+h) + \sqrt{x+h} - (x + \sqrt{x})}{h}$$

$$= \lim_{h \to 0} \frac{x + h - x + \sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} + \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} 1 + \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} 1 + \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} 1 + \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} 1 + \frac{1}{\sqrt{x+h} + \sqrt{x}} = 1 + \frac{1}{2\sqrt{x}}.$$

3. $$y = \frac{x^2 - 2\sqrt{x}}{x} = x - 2x^{-1/2}.$$

So $$\frac{dy}{dx} = \frac{d}{dx}(x) - 2\frac{d}{dx}\left(x^{-1/2}\right)$$

$$= 1 - 2 \cdot \left(\frac{-1}{2} x^{-3/2}\right)$$

$$= 1 + x^{-3/2}.$$

4. $y = \csc x = \dfrac{1}{\sin x}$.

By the quotient rule, $\dfrac{dy}{dx} = \dfrac{(\sin x)\frac{d}{dx}(1) - 1\cdot\frac{d}{dx}(\sin x)}{(\sin x)^2}$

$$= \frac{(\sin x)\cdot 0 - 1\cdot\cos x}{(\sin x)^2}$$

$$= \frac{-\cos x}{(\sin x)^2}.$$

(or $-\cot x \cdot \csc x$, the same thing)

5. $y = \sec^2 x + \tan^2 x$

Sol'n 1: $\sec^2 x + \tan^2 x = \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}$

$$= \frac{1+\sin^2 x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{(\cos^2 x)\frac{d}{dx}(1+\sin^2 x) - (1+\sin^2 x)\frac{d}{dx}(\cos^2 x)}{\cos^2 x}.$$

What is $\frac{d}{dx}(\sin^2 x)$? If $u = \sin x$,

$$\frac{d}{dx}(\sin^2 x) = \frac{du^2}{dx} = \frac{du^2}{du}\frac{du}{dx}$$

$$= 2u\cdot\frac{du}{dx}$$

$$= 2\sin x\cdot\cos x,$$

Similarly $\frac{d}{dx}(\cos^2 x) = 2\cdot\cos x\cdot\frac{d}{dx}(\cos x)$

$$= -2\cos x\cdot\sin x.$$

(cont.)

So,

$$\frac{dy}{dx} = \frac{(\cos^2 x)\cdot 2\sin x\cos x - (1+\sin^2 x)\cdot(-2\sin x\cos x)}{(\cos^2 x)^2}$$

$$= \frac{2\cdot\sin x\cos x\,(\cos^2 x) + 2\cdot\sin x\cos x + 2\cdot\sin x\cos x\,(\overset{\sin^2 x}{\cancel{\cos^2 x}})}{\cos^4 x}$$

$$= \frac{2\cdot\sin x\cos x\,(\cos^2 x + \sin^2 x) + 2\sin x\cos x}{\cos^4 x}$$

$$= \frac{2\cdot\sin x\cos x + 2\cdot\sin x\cos x}{\cos^4 x}$$

$$= \frac{4\cdot\sin x\cos x}{\cos^4 x} = \frac{4\cdot\tan x}{\cos^2 x}.$$

Solution 2: We know $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$

$$= \frac{(\cos x)\cdot 0 - 1\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x\cdot\cos x - \sin x\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}.$$

So, $\frac{d}{dx}(\sec^2 x + \tan^2 x)$

$$= 2 \cdot \sec x \cdot \frac{d}{dx}(\sec x) + 2 \cdot \tan x \cdot \frac{d}{dx}(\tan x)$$

$$= 2 \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos^2 x} + 2 \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^2 x}$$

$$= 4 \cdot \frac{\sin x}{\cos^3 x} = 4 \cdot \frac{\tan x}{\cos^2 x}.$$

6. $\frac{d}{dx}(e^{x/y}) = \frac{d}{dx}(x - y)$

$$e^{x/y} \frac{d}{dx}\left(\frac{x}{y}\right) = 1 - \frac{dy}{dx} \quad \text{(Chain rule)}$$

$$e^{x/y} \cdot \frac{y \cdot \frac{dx}{dx} - x \cdot \frac{dy}{dx}}{y^2} = 1 - \frac{dy}{dx}$$

$$e^{x/y} \cdot \frac{y - x \cdot \frac{dy}{dx}}{y^2} = 1 - \frac{dy}{dx}$$

$$\frac{e^{x/y}}{y} - \frac{xe^{x/y}}{y^2}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

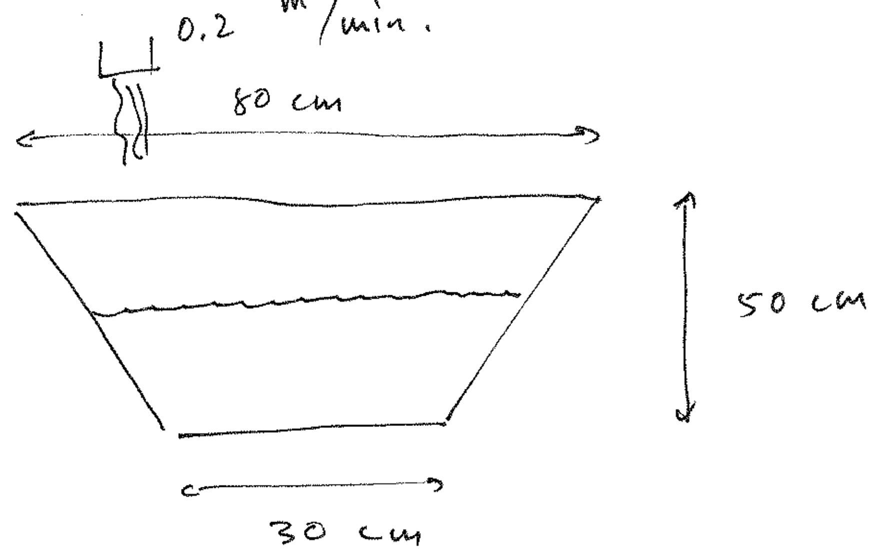
$$\frac{e^{x/y}}{y} - 1 = \frac{xe^{x/y}}{y^2}\frac{dy}{dx} - \frac{dy}{dx}$$

$$\frac{e^{x/y}}{y} - 1 = \left(\frac{xe^{x/y}}{y^2} - 1\right)\frac{dy}{dx}$$

So, $\dfrac{dy}{dx} = \dfrac{\dfrac{e^{x/y}}{y} - 1}{\dfrac{x e^{x/y}}{y^2} - 1} = \dfrac{\dfrac{e^{x/y} - y}{y}}{\dfrac{x e^{x/y} - y^2}{y^2}}$

$= \dfrac{y(e^{x/y} - y)}{x e^{x/y} - y^2}$.

7.

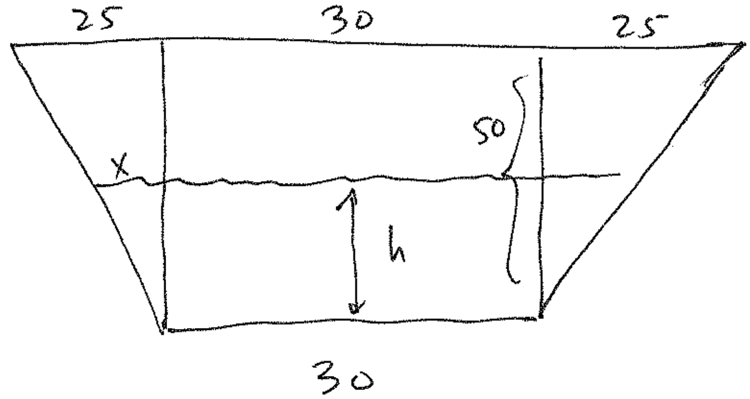


Let : t = time in seconds
h = height of water
V = volume of water.

Then $\dfrac{dV}{dt} = 0.2 \ m^3/min = 200{,}000 \ cm^3/min$.

Need a relation between V and h.

Volume = (10 m) × (area of cross-section)
= (1000 cm) × (area).



We know $\text{Area} = h \cdot 30 + \frac{1}{2} \cdot x \cdot h + \frac{1}{2} \cdot x \cdot h$

$$= 30h + xh.$$

By similar triangles, $\frac{x}{h} = \frac{25}{50}$, so $x = \frac{h}{2}$.

So $\text{Area} = 30h + \frac{h^2}{2}$.

So $V = 1000\left(30h + \frac{h^2}{2}\right)$

$$= 30000h + 500\frac{h^2}{2}.$$

Therefore $\frac{dV}{dh} = 30000 + 1000h$.

Now $\frac{dh}{dt} \cdot \frac{dV}{dh} = \frac{dV}{dt}$,

so $\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}} = \frac{200000}{30000 + 1000h}$.

Plug in $h = 30$, $\frac{dh}{dt} = \frac{200000}{30000 + 30000}$

$$= \frac{200000}{60000} \text{ cm/min}$$

$$= \frac{20}{6} = \frac{10}{3} \text{ cm/min}.$$

8. We know $\dfrac{d}{dx}(\ln|x|) = \dfrac{1}{x}$.

If $x > 0$ then $|x| = x$ and $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

If $x < 0$ then $|x| = -x$ and $\dfrac{d}{dx}(\ln(-x)) = \dfrac{1}{-x} \cdot \dfrac{d}{dx}(-x)$

$$= \frac{-1}{-x} = \frac{1}{x}.$$

So: $\dfrac{d}{dx}\left(\ln|\sec 5x + \tan 5x|\right)$

$$= \frac{1}{\sec 5x + \tan 5x} \cdot \frac{d}{dx}(\sec 5x + \tan 5x)$$

$$= \frac{1}{\sec 5x + \tan 5x}\left(\frac{\sin 5x}{\cos^2 5x} \cdot \frac{d}{dx}(5x) + \frac{1}{\cos^2 5x} \cdot \frac{d}{dx}(5x)\right)$$

$$= \frac{1}{\sec 5x + \tan 5x}\left(\frac{5(\sin 5x + 1)}{\cos^2 5x}\right).$$

We can simplify!

This is $\dfrac{1}{\dfrac{1}{\cos 5x} + \dfrac{\sin 5x}{\cos 5x}} \cdot \dfrac{5(\sin 5x + 1)}{\cos^2 5x}$

$$= \frac{\cancel{1}\cos 5x}{1 + \sin 5x} \cdot \frac{5(\sin 5x + 1)}{\cos^2 5x}$$

$$= \frac{5 \cdot \cos 5x}{\cos^2 5x} = \frac{5}{\cos 5x} = 5 \sec 5x.$$

9. Here is one such graph.