Homework 2 - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, September 2

As always, please show your work and explain yourself clearly.

(a) Suppose that you are given the graph of the function $y = f(x)$, and you want to find the slope of the tangent line to the graph at the point $(a, f(a))$. Explain how to guess this slope by finding the slopes of secant lines.

(b) Given the slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$, explain how to find the equation of this line.

(c) By finding the slopes of appropriate secant lines, determine the equation of the tangent line to $y = x^2$ at $(1, 1)$.

(d) By finding the slopes of appropriate secant lines, determine the equation of the tangent line to $y = -x$ at $(4, -4)$.

(e) By finding the slopes of appropriate secant lines, determine the equation of the tangent line to $y = x^3 - 3$ at $(1, -2)$.

(f) Stewart, Ch. 2.2, 1-4, 7, 9, 13.

(g) Stewart, Ch. 2.3, 11-30, 40, 42.

For this part only, the even problems are required and the odd problems are strongly recommended.

(h) Consider the following definition of a limit: 'We say that $\lim_{x \to a} f(x) = c$ if $f(x)$ gets closer and closer to c as x gets closer and closer to a.'

What is wrong with this definition?

(i) Give the definition of a limit, e.g., explain what it means to say that $\lim_{x \to a} f(x) = c$. You may explain in words, or using the $\epsilon - \delta$ formalism.

(j) Explain what it means for a function $f(x)$ to be continuous. (The informal definition is okay.)

(k) Stewart, Ch. 2.5, 7-8.

(l) Graph the following functions. Which are continuous? For the functions that are not continuous, explain why.

$f(x) = x^2$, $f(x) = \sin(x)$, $f(x) = \frac{1}{x-2}$, $f(x) = \frac{x-1}{x^2-1}$, $f(x) = e^x$.