Homework 11 - Math 141, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, November 4

For related rates and optimization problems: Please draw and label your pictures, list and describe your variables, highlight any equations that occur, and explain all of your work.

Some of these problems are borrowed from `http://www.ms.unimelb.edu.au/~ram/Teaching/2006Fallmath221/HW8F06.pdf` but you don't need to look there unless you're curious.

Related rates problems:

(a) Find the rate of the change of the volume of a sphere with respect to a change in the radius.

(b) Find the rate of change of the volume of a cylinder of height 1 with respect to a change in the radius.

(c) Find the rate of change of the volume of a cylinder of radius 1 with respect to a change in the height.

(d) The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of change of the perimeter of the square.

(e) The bottom of a rectangular swimming pool is 30×50 feet. Water is pumped into the tank at the rate of 500 cubic feet per minute. Find the rate at which the level of the water in the tank is rising.

(f) A five foot tall woman is standing at some distance from a 20 foot tall pole, and is walking away from the pole. A streetlight is on top of the pole. How fast is the tip of her shadow moving as she walks away from the pole?

(g) A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

(h) A boat is pulled into a dock by a rope attached to the bow of the boat, passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8 m from the dock?

(i) A 10 foot ladder rests on a wall and is sliding down the wall (and the bottom is sliding away from the wall). If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?

(j) A television camera is positioned 4000 ft from the base of a rocket launching pad. A rocket rises vertically, and its speed is 600 ft/s when it has risen 3000 feet. How fast is the distance from the television camera to the rocket changing at that moment?

(k) In the previous question, how fast is the camera's angle of elevation changing at that same moment?

Other problems:

(l) Find two numbers whose sum is 10 and whose product is a maximum.

(m) Find two numbers whose sum is 10 and whose product is a minimum. (**This is not possible! Explain why.**)

(n) Find two numbers whose difference is 10 and whose product is a minimum.

(o) A farmer wants to build a rectangular fence along a river. She has 100 feet of fencing, and she does not need any fence along the river. What is the most area she can enclose?

(p) Stewart, Ch. 4.7: 11, 12, 14, 15, 17, 18, 21, 30, 44, 46, 47, 50, 52, 55, 58, 63 (all).

(q) Stewart, Ch. 4.9, 1-14 (odd recommended; even required)