Bonus Homework and Exam Review - Math 141, Frank Thorne
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Due Friday, December 9 at the start of the final exam.

It is strongly recommended that you do as many of these problems as possible. This is the best way to review for the exam. I recommend the following: Work on ten problems at a time, and come up with solutions without checking the book or your notes. Take a break. Then, look at your solutions and compare them to the book, your notes, and your old homeworks and see if there is anything you want to change.

For any graphing problems: Find all critical points, inflection points, intervals of increase and decrease, where the graph is concave up or down, and horizontal and vertical asymptotes.

Where appropriate: Always draw a diagram and/or a graph if it is relevant to your problem, and explain yourself clearly.

The exam will include, among other questions: at least one graphing question, at least one optimization question, at least one related rates question, at least one volume or area question, at least one question where you are given a graph and have to say something about it, and at least one of the first eight questions below.

Extra credit: This assignment is completely optional. It is worth extra credit equivalent to two homework assignments. **Partial extra credit cheerfully given**; this is *extremely long*, if you get through part of this, please turn in what you have.

Good luck! Taylor, Lauren, and I are all here to help if you get stuck.

(1) Give the definition of the derivative of a function $f(x)$ at the point $x = a$. (Please give an algebraic definition, using an equation.) Draw a picture and explain why your equation gives the slope of the tangent line to the graph of $f(x)$ at $x = a$.

(2) Give the definition of the definite integral of a function $f(x)$ between $x = a$ and $x = b$. (Give an algebraic definition, using an equation.) Draw a picture and explain why your equation gives the area under the graph of $f(x)$ between a and b.

(3) Give the definition of the indefinite integral of a function $f(x)$.

(4) What do indefinite integrals have to do with derivatives, and how do you know?

(5) What do definite integrals have to do with derivatives, and how do you know?

(6) State both parts of the Fundamental Theorem of Calculus.

(7) The first part of the Fundamental Theorem of Calculus tells you that $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$. Why is this true? Draw a picture and explain.

(8) What does the Mean Value Theorem say? Draw a picture and explain.

(9) Find the absolute maximum and minimum of $f(x) = xe^{-x^2/8}$ on the interval $[-1, 4]$.

(10) If f and g are two functions, draw a picture which explains why $(fg)' = f'g + g'f$.

(11) A snowball melts so that its surface area decreases at a rate of 1 cm^2/min. Find the rate at which the diameter decreases when the diameter is 10 cm. (The surface area of a sphere of radius r is $4\pi r^2$.

(12) Find the absolute maximum and minimum values of $f(x) = \frac{x}{x^2+1}$ on $[0, 2]$.

(13) Give an equation of a function which is not differentiable (at at least one point), and explain why it is not differentiable.

(14) Do prob. 7 on p. 285 of Stewart.

(15) Find $f'(t)$ if $f(t) = \frac{1}{2}t^6 - t^4 + t$.

(16) Define the function $\cos(x)$.

(17) A man starts walking north at 4 ft/s from a point P. Five minutes later a woman starts walking south at 5 ft/s from a point 500 ft due east of P. At what rate are they moving apart 15 min after the woman starts walking?

(18) Do prob. 3 on p. 162 of Stewart.

(19) Verify that $f(x) = \sqrt{x} - x/3$ satisfies the hypotheses of Rolle's thorem on the interval $[0, 9]$. Then find all numbers c that satisfy the conclusion of Rolle's theorem.

(20) Find the absolute maximum and minimum of $f(t) = 2\cos t + \sin 2t$ on the interval $[0, \pi/2]$.

(21) Do prob. 6 on p. 162 of Stewart.

(22) Graph a function f for which $f(0) = 0$, $f(1) = 1$, $\lim_{x\to\infty} f(x) = 0$, and f is odd.

(23) Find all local and absolute maxima and minima of $f(x) = e^x$.

(24) Find $\int_0^4 \sqrt{4x - x^2}$ using geometry.

(25) Find $f'(u)$ if $f(u) = \frac{\ln u}{1+\ln(2u)}$.

(26) Find the derivative of $f(x) = \sqrt{1+2x}$ using the definition of the derivative. State the domain of $f(x)$ and of its derivative.

(27) Do prob. 9 on p. 97 of Stewart.

(28) Do prob. 61 on p. 297 of Stewart.

(29) Graph the function $y = x^3$ on $[0, 1]$ and estimate the area under the curve by dividing it into four intervals.

(30) Find the first and second derivatives of $G(r) = \sqrt{r} + r^{1/3}$.

(31) What does $\lim_{x\to-\infty} g(x) = 6$ mean?

(32) Find y' if $y = x^x$.

(33) Explain why $\frac{d}{dx}(\ln x) = \frac{1}{x}$. (You may use any fact you know about the function $y = e^x$.

(34) A Norman window has the shape of a rectangle with a semicircle on top. If the perimeter of the window is 30 ft, find the dimensions of the window with the largest area.

(35) Graph $f(x) = e^{-x^2}$.

(36) Find $\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$.

(37) Find y' if $y = \frac{x+1}{x^3+x-2}$.

(38) If $f(x) = 3x^2 - 5x$, find $f'(2)$ and use it to find an equation of the tangent line to the parabola $y = 3x^2 - 5x$ at the point $(2, 2)$.

(39) Do prob. 52 on p. 345 of Stewart.

(40) Find the derivative of $f(t) = 5t - 9t^2$ using the definition of the derivative. State the domain of the function and of its derivative.

(41) Sketch the graph of a function with two local maxima, one local minimum, and no absolute maximum.

(42) Do prob. 4 on p. 388 of Stewart.

(43) Find $g'(x)$ if $g(x) = \frac{3x-1}{2x+1}$.

(44) Find the area bounded by the curves $y = 1/x$, $y = 1/x^2$, and $x = 2$.

(45) Graph $f(x) = \frac{x}{x-1}$.

(46) If $\sqrt{xy} = 1 + x^2y$, find $\frac{dy}{dx}$. (Your answer may have y in it.)

(47) Find $\lim_{x\to -\infty} \frac{1}{2x-3}$.

(48) Do prob. 42 on p. 164 of Stewart.

(49) A farmer wants to build a rectangular fence along a river. She has 100 feet of fencing and does not need any fence along the river. How much area can she enclose?

(50) Find the limit $\lim_{x\to -2} \frac{2-|x|}{2+x}$ if it exists. If it does not exist, say why.

(51) Evaluate $\int_0^1 10^x dx$.

(52) A boat leaves a dock at 2:00 and travels due south at 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00. At which time were the two boats closest together?

(53) Find the volume of the solid obtained by rotating the region bounded by $y = 1/x$, $x = 1$, $x = 2$, $y = 0$ about the x-axis.

(54) Let $f(x) = x^2$. Use the Mean Value Theorem to show that there is a point $c \in (-1, 2)$ with $f'(c) = 1$. Then, find $f'(x)$ and figure out all such values of c. Draw a picture which illustrates your conclusions.

(55) Explain why $\frac{d}{dx}(\cos x) = -\sin x$.

(56) Find the derivative of $f(x) = \csc x$ and $g(x) = \sec x$.

(57) Find the volume of a square pyramid with base length b and height b.

(58) Find the point on the line $y = 4x + 7$ that is closest to the origin.

(59) Find $\lim_{x\to\infty} (\sqrt{9x^2 + x} - 3x)$.

(60) If $1 + x = \sin(xy^2)$, find $\frac{dy}{dx}$. (Your answer may have y in it.)

(61) Find the volume of a sphere with radius r.

(62) A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs 10 dollars per square meter. Material for the sides costs 6 dollars per square meter. Find the cost of materials for the cheapest such container.

(63) Find the volume of the solid obtained by rotating the region bounded by the following curves about the y-axis: $y^2 = x$, $x = 2y$.

(64) Graph $f(x) = \frac{x}{x^2+9}$.

(65) You drive 120 miles from Columbia to Charlotte, and the trip takes two hours. At some point, your speed is exactly 60 mph. Explain how you know this. Must there be a point at which your speed is exactly 65 mph?

(66) The number of bacteria after t hours in a laboratory experiment is $n = f(t)$. What is the meaning of the derivative $f'(5)$? What are its units?

Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? Does it matter if the supply of nutrients is limited? Explain.

(67) Do prob. 7 on p. 410 of Stewart.

(68) Explain why $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ without L'Hospital's rule.

(69) Graph $y = x\sqrt{5 - x}$.

(70) Find the limit $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}}$.

(71) Find $\frac{dy}{dx}$ if $y = \sin^{-1} x$.

(72) Evaluate $\int_0^1 xe^{-x^2} dx$.

(73) A television camera is positioned 4000 ft from the base of a rocket launching pad. A rocket rises vertically, and its speed is 600 ft/s when it has risen 3000 feet. How fast is the distance from the television camera to the rocket changing at that moment?

(74) Find two numbers whose sum is 10 and whose product is a maximum.

(75) Find $\lim_{x\to\infty} \cos x$.

(76) Graph $y = x\tan x$ for $-\pi/2 < x < \pi/2$.

(77) Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the pile increasing when the pile is 10 ft high?

(78) You and nine friends are working on calculus problems together. Each of you can solve problems at a rate of $r(t) = 6 + t$, where t is the number of hours you have worked (you improve as you go along!), and $r(t)$ is the number of calculus problems per hour.

If each problem takes half a page, and you work front and back, and a piece of paper is 0.01 in thick, how fast is the pile of homework problems increasing when you have been working for 35 hours?

(79) Evaluate $\int \frac{\sin 2x}{\sin x} dx$.

(80) Evaluate $\int_0^7 \sqrt{4+3x}dx$.

(81) Graph $y = x/2 - \sin x$ for $0 < x < 3\pi$.

(82) A parking lot charges \$5 for the first hour (or part of an hour) and \$5 for each succeeding hour (or part), up to a daily maximum of \$20. Sketch a graph of the cost of parking at this lot as a function of the time parked there. In addition, discuss the discontinuities of this function and their significance to someone who parks in the lot.

(83) Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

(84) Find y' if $y = \frac{1-\sec x}{\tan x}$.

(85) Find the horizontal and vertical asymptotes of $y = \frac{x^3-x}{x^2-6x+5}$.

(86) Evaluate $\int \cos\theta \sin^6 \theta d\theta$.

(87) Graph $y = x - \ln x$.

(88) A lighthouse is on a small island 3 km away from the nearest point P on a straight shoreline and its light turns four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

(89) Find the derivative of $y = (2x-5)^4(8x^2-5)^{-3}$.

(90) Evaluate $\int e^x \sin(e^x)dx$.

(91) A trough is 10 ft long and its ends have the shape of isoceles traingles that are 3 ft across the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

(92) Graph $y = 1/(1 + e^{-x})$.

(93) Explain what it means to say that $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 7$. In this situation is it possible that $\lim_{x\to 1} f(x)$ exists?

(94) A ball is thrown into the air with velocity 40 ft/s, and its height in feet after t seconds is given by $y = 40t - 16t^2$. Find the velocity when $t = 2$.

(95) Evaluate $\int x \sin(x^2) dx$.

(96) Find the equation of the tangent line to $y = \frac{1}{\sin x + \cos x}$ at the point $(0, 1)$.

(97) Find the derivative of $f(x) = (1 + 2x + x^3)^{1/3}$.

(98) Graph $y = \frac{x^2-4}{x^2-2x}$. (Hint: look for a partial shortcut)

(99) Do prob. 17 on p. 151 of Stewart.

(100) Find $\lim_{\theta\to 0} \frac{\tan 6\theta}{\sin 2\theta}$.