Final Examination (version 6) - Math 141, Frank Thorne (thorne@math.sc.edu)

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Please work without books, notes, calculators, or any assistance from others. Please **show all your work, explain yourself clearly, draw pictures where appropriate, and put equals signs where they belong.**

If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

Each problem is worth 10 points; a total of 160 points is possible. **GOOD LUCK!**

(1) What is a definite integral? Explain thoroughly and draw a picture.

Clarification: Your answer should include the precise definition of the definite integral given in class, or alternatively that given in the book.

(2) Give an equation of a function which is not differentiable, and algebraically explain why it is not differentiable.

(3) Compute

$$\lim_{y \to 2} \frac{y+2}{y^2+5y+6}.$$

(4) Say whether the function graphed in Figure 1 is continuous on $[-1, 3]$. If not, where does it fail to be continuous and why?

(5) Find the slope of the graph of the function $h(t) = t^3 + 3t$ at the point $(1, 4)$. Then find an equation for the line tangent to the graph there.

For this problem, use the definition of the derivative at a point, and do not use differentiation rules such as the power, product, or quotient rules.

(6) Find the derivative of the function

$$w = re^{-r}.$$

(7) Find $\frac{dy}{ds}$, if

$$y = 2^{(s^2)}.$$

(8) Compute $\sec^{-1}(-\sqrt{2})$, $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$, and $\sec^{-1}(-2)$.

(9) The equation

$$s = \frac{25}{t^2} - \frac{5}{t}, \quad 1 \leq t \leq 5$$

gives the equation of a body moving on a coordinate line, with s in meters and t in seconds.

(a) Find the body's displacement and average velocity for the given time interval.

(b) Find the body's speed and acceleration at the endpoints of the interval.

(c) When, if ever, during the interval does the body change direction?

(10) Two commercial airplanes are flying at an altitude of $40,000$ ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 442 knots (nautical miles per hour; a nautical mile is 2000 yd). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 nautical miles from the intersection point and B is 12 nautical miles from the intersection point?

Since you don't have access to a calculator, you are welcome to approximate messy numbers. If you choose to do so, explain clearly what you are doing.

(11) Graph the function

$$y = 1 - 9x - 6x^2 - x^3$$

according to the following instructions taken from the book:

Procedure for graphing $y = f(x)$:

(a) Identify the domain of f and any symmetries the curve may have.
(b) Find the derivatives y' and y''.
(c) Find the critical points of f, if any, and identify the function's behavior at each one.
(d) Find where the curve is increasing and where it is decreasing.
(e) Find the points of inflection, if any occur, and determine the concavity of the curve.
(f) Identify any asymptotes that may exist.
(g) Ploy key points, such as the intercepts and the points found in steps (c)-(e), and sketch the curve together with any asymptotes that may exist.

(12) The positions of two particles on the s-axis are $s_1 = \sin t$ and $s_2 = \sin(t + \pi/3)$, with s_1 and s_2 in meters and t in seconds.

(a) At what time(s) in the interval $0 \leq t \leq 2\pi$ do the particles meet?
(b) What is the farthest apart that the particles get?

(13) Evaluate the integral

$$\int (1 - x^2 - 3x^5)\, dx.$$

(14) Evaluate the integral

$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx.$$

(15) Find the total area between the region given by

$$y = x^{1/3} - x, \quad -1 \leq x \leq 8$$

and the x-axis.

(16) Find the volume of the solid generated by revolving the region bounded by the following lines and curves about the x-axis:

$$y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1.$$