Math 511
Review Exam 1 Solutions

1. (a) 0.8 (b) 0.84 (c) 0 (d) 0.944 (e) 0.2286 (f) 0.87 (g) 0.25 (h) 0.9

2. \[
\binom{13}{2} \binom{4}{2}^2 \binom{11}{3} \binom{4}{2}^2 \binom{52}{12}
\]

3. \[
\binom{7}{2} \binom{8}{3}^2 \binom{5}{1} \binom{8}{4} \binom{8}{4} \binom{15}{5} \binom{56}{15}
\]

4. Let \( A \) be the event that the first six draws contains exactly two reds, and let \( B \) be the event that the 7\(^{th} \) draw is red. Then we want \( P(A \cap B) = P(A \mid B)P(B) \).

Without replacement this is: \( 0.2 \times \frac{\binom{4}{2} \binom{12}{4}}{\binom{16}{6}} \).

With replacement, we get \( \frac{1}{4} \cdot \left( \frac{\binom{6}{2} \binom{1}{4}^2 \binom{3}{4}}{\binom{4}{4}} \right)^4 \).

5. Let \( KK \) denote the event that we draw two Kings and let \( ALOK \) denote the event that we draw At Least One King. Then,

\[
P(KK \mid ALOK) = \frac{P(ALOK \cap KK)}{P(ALOK)} = \frac{P(ALOK \mid KK)P(KK)}{P(ALOK)}
\]

\[
= \frac{P(KK)}{P(ALOK)} = \frac{\binom{4}{2}/\binom{8}{2}}{1 - \binom{4}{2}/\binom{8}{2}} = \frac{\binom{4}{2}/\binom{8}{2}}{\binom{8}{2} - \binom{4}{2}} = \frac{6}{22} = \frac{3}{11}
\]
6. \( P(X \mid 2 \text{ of } 6 \text{ are bad}) = \frac{P(2 \text{ of } 6 \text{ are bad} \mid X)P(X)}{P(2 \text{ of } 6 \text{ are bad})} = \)

\[ \frac{\binom{10}{2} \binom{30}{4} \frac{1}{2}}{\binom{40}{6}} = \frac{\left[ \binom{10}{2} \binom{30}{4} \frac{1}{2} + \binom{8}{2} \binom{20}{4} \frac{1}{2} \right]}{\binom{40}{6} + \binom{28}{6}} \]

7. \( 1 - \frac{(n-1)(n-2)(n-3)\ldots(n-99)}{n^{99}} \)

8. \( P(W) = P(W \text{ and } \cap W \text{ transferred}) + P(W \text{ and } \cap L \text{ transferred}) = \)\[
\begin{align*}
P(W) &= P(W \mid W \text{ transferred})P(W \text{ transferred}) + P(W \mid L \text{ transferred})P(L \text{ transferred}) \\
&= (0.4)(0.4) + (0.3)(0.6) = 0.34
\end{align*}
\]

\[ \frac{P(W \text{ transferred} \mid W \text{ drawn})}{P(W \text{ drawn})} = \frac{P(W \text{ drawn} \mid W \text{ transferred})P(W\text{transferred})}{P(W \text{ drawn})} = \frac{0.16}{0.34} = \frac{8}{17} \]
9. There are 11 odd-numbered balls and 10 that are even-numbered. The sum of two numbers is even if they are both even or both are odd. So we get,
\[
\binom{11}{2} + \binom{10}{2} = \frac{10}{21}.
\]
If three balls are drawn, then we get
\[
\binom{10}{3} + 10 \binom{11}{2} = \frac{67}{133}.
\]

10. (a) \[
\frac{6!}{3!3!} = \frac{1}{84}\]
(b) \[
\frac{3!6!}{2!2!2!} \approx 0.321
\]

11. \[
\left(\frac{12}{4}\right)\left(\frac{1}{2}\right)^{12}
\]

12. \[
1 - \left(1 - \frac{1}{n}\right)^{3n} = 1 - \left[\left(1 - \frac{1}{n}\right)^n\right]^3 = 1 - \frac{1}{e^3} \text{ (when } n \text{ is large)}.
\]

What if each of 200 boxes contains 3 red balls and 97 green balls and a single ball is drawn out of each of the boxes. What is the probability that at least one of the balls is red? Answer:
\[
1 - \left(1 - \frac{3}{100}\right)^{200} = 1 - \left[\left(1 - \frac{3}{100}\right)^{100}\right]^2 \approx 1 - \left(\frac{1}{e^3}\right)^2 = 1 - \frac{1}{e^6} = 0.997!
\]

13. (a) \[
1 - (0.4)^4 = 0.9744
\]
(b) \[
\binom{4}{2}(0.4)^2(0.6)^2 = 0.3456
\]
14. \[
\left( \frac{20}{3} \right) \left( \frac{17}{3} \right)^{4^{14}} \div 6^{20}
\]

15. \[
1 - \left( \frac{7}{4} \right)^{4^{4}}
\]

16. (a) \[
\frac{3 \cdot 9! + 7 \cdot 6 \cdot 3 \cdot 7! + 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 5! + 7! \cdot 3 \cdot 3!}{10!}
\]

(b) \[
\frac{0.30}{1 - 0.49} = \frac{10}{17}
\]

17. \[
\frac{8!}{3!2!3!} \div 10^8
\]