

**Math 511**  
**Review Exam #1**

Of course, you should go over your notes and all the problems assigned on problem sets 1 – 9 or discussed in class.

Some other problems from your text:

Page 79 – 82 # 9, 10, 20      Page 89 – 92 # 5, 17      Page 98 – 100 # 7, 10, 18  
Page 103 – 104 # 2

In addition, here are some extra problems to consider.

- Suppose that  $A, B, C, D, E$  and  $F$  are events in a sample space  $S$ . Suppose too that  $P(A) = 0.3, P(B) = 0.4, P(C) = 0.7, P(D) = 0.2, P(E) = 0.35, P(F) = 0.8$ . Moreover,  $D, E$  and  $F$  are mutually independent, and  $C$  and  $D$  are mutually exclusive.  $P(A | B) = 0.6$ .  $P(A \cup D) = 0.45$ . Determine the value of:  
(a).  $P(B | A)$ ?    (b).  $P(A \cup B')$     (c).  $P(C \cap D)$       (d).  $P(D' \cup E' \cup F')$   
(e).  $P(B | A')$     (f).  $P(E \cup F)$     (g).  $P(A | D)$       (e).  $P(C \cup D)$
- If 12 cards are drawn at random from a standard deck of 52, what is the probability that they consist of 3 each of two kinds and 2 each of each of three additional kinds? (Example: A A A J J J 8 8 7 7 4 4)
- Suppose that a box contains 8 each of red, blue, green, yellow, white, black, and orange marbles (so 56 marbles altogether). If you choose 15 marbles at random, what is the probability that you choose 3 each of two separate colors and 4 of a third color and 5 of a fourth color?
- A box contains 4 red balls and 12 green balls. Balls are chosen one at a time without replacement. What is the probability that the 3<sup>rd</sup> red marble appears on draw 7? (What would the answer be if the balls were drawn with replacement?)
- Two cards are drawn at random from a 'deck' consisting of 4 Kings and 4 Jacks. You are told that at least one of the cards is a King. What is the probability that they are both Kings?
- Suppose that box X contains 30 good light bulbs and 10 bad light bulbs. While box Y contains 20 good bulbs and 8 bad bulbs. Suppose that you randomly choose one of the boxes and pick 6 bulbs without replacement from the box. Given that exactly two of them are bad, what is the probability that you chose box X?
- Suppose that each person in a room of 100 people chooses a number from 1 –  $n$  at random ( $n > 100$ ). Determine an expression for the probability that at least two people choose the same number.

8. Suppose that a box contains 4 cards labeled W (for win) and 6 cards labeled L (for lose). A second box contains 3 cards labeled W and 6 cards labels L. A card is picked at random form the first box and then placed in the second box. After this, a card is selected at random from the second box.
  - (a). What is the probability that this card is labeled W?
  - (b). Given that the card you drew was marked W, what is the probability that the card that was transferred was also labeled W?
  
9. A box contains 21 Ping-Pong balls distinctly numbered from 1 – 21. If two balls are chosen at random, what is the probability that their sum is even? What is the answer if three balls are chosen instead of two?
  
10. Each possible arrangements of 3 A's, 3 B's and 3 C's is written on a card and all the cards are then placed into a hat. A card is chosen at random.
  - (a).What is the probability that the first three letters on the card are all A's?
  - (b). What is the probability that the first three letters on the card are all different?
  
11. If 12 coins are flipped at random, what is the probability that exactly 4 of them show heads?
  
12. Suppose that an n-sided die is rolled 3n separate times. What is the exact probability that the number n appears at least once. What is this approximately equal to if n is very large? (Compare this to 2.4.18 page 100 of your text).
  
13. Suppose that  $A_1, A_2, A_3,$  and  $A_4$  are mutually independent events in the same sample space and that each of these events has the probability 0.6 of occurring.
  - (a). What is the probability that at least one of these occurs?
  - (b). What is the probability that exactly two of the events occur?
  
14. If 20 distinct marbles are placed at random into 6 distinct boxes, what is the probability that the first two boxes contain exactly three marbles each?
  
15. A box contains 4 red, 4 blue, 4 green, 4 yellow, 4 white, 4 purple, and 4 black jelly beans. Suppose that four jellybeans are chosen at random and without replacement. What is the probability that at least two of them are the same color?
  
16. A box contains 3 cards marked W and 7 cards marked L. Two people play a game where each in turn chooses a card at random. The first player to get a card marked W wins. What is the probability that the first player wins if the cards are chosen
  - (a).without replacement? (b).with replacement?
  
17. Suppose that a lottery consists of choosing 8 digits with replacement. So one possible choice would be 5 4 7 7 2 4 6 7. You also choose 8 digits and if any permutation of your choice matches that of the lottery's choice, then you win. So if you chose 4 4 2 6 7 7 7 5 you'd win the example drawing. What is the probability that 1 1 1 2 2 3 3 6 is a winning ticket?