Math 547–Review Problems–Exam 1

Be able to do problems such as those below and the problems assigned on problem sets or the following problems from your textbook.

Textbook Problems:
Page 110–111: 1, 2, 3, 4, 7, 9, 13 14(c), 15, 18, 20.
Page 174–176: 1, 5, 8, 11, 13, 14, 15, 29, 30, 47, 50, 55.
Page 182–184: 1, 2, 5, 9, 11, 14, 15, 16, 29.
Page 326–327: 1, 2, 3, 5, 7, 9, 13.

1. The center of a ring $R$ is the set $Z(R) = \{a : ar = ra \text{ for all } r \in R\}$.
   Show that the center of a ring is a subring of the ring.

2. What are the units of the polynomial ring $\mathbb{R}[x]$ where $\mathbb{R}$ is the set of real numbers?

3. Find an example of a ring and two elements $a$ and $b$ such that $ab = 0$ but $ba \neq 0$.

4. Suppose that $R$ is a ring with identity and $a$ is an element of $R$ such that $a^2 = 1$.
   Is $S = \{ara : r \in R\}$ a subring of $R$?

5. Is $S = \left\{ \begin{bmatrix} a & a+b \\ a+b & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}$ a subring of $M_2(\mathbb{Z})$?

6. Is $S = \left\{ \begin{bmatrix} a & a \\ b & b \end{bmatrix} : a, b \in \mathbb{Z} \right\}$ a subring of $M_2(\mathbb{Z})$?

7. Show that if $A$ and $B$ are ideals of a ring $R$, then $A + B = \{a+b : a \in A, b \in B\}$ is an ideal of $R$.

8. Suppose that an ideal $A$ of a ring $R$ contains a unit, then show that $A = R$.

9. What are the possible ideals of a field?

10. Show that every group of order 35 is cyclic.

11. What are the units of $\mathbb{Z}_{24}$?

12. (a) If $R$ is a ring of characteristic 3, then simplify the expansion of $(a+b)^3$.
    (b) If $R$ is a ring of characteristic 4, then simplify the expansion of $(a+b)^5$. 