Math 242, Section E01 Name Key

Short Quiz No. 8 March 22, 2017

Show all work, and if your solution involves several steps, place a $\boxed{\text{box}}$ around your final answer. Answer **five** of the questions I.–VI. Questions V. and VI. are on the back of this page.

I. (2) Find a formula for the polynomial differential operator L for which the differential equation $L(y) = \cos 2x$ is the differential equation $y^{(6)} - 8y^{(3)} = \cos 7x$, and then factor L as much as possible.

$$\boxed{L = D^6 - 8D^3}$$
$$= D^3(D-8)$$
$$= \boxed{D^3 \cdot (D-2) \cdot (D^2+2D+4)}$$

a root of $r^3 - 8 = 0$ is $r = 2$, and

$$\begin{array}{r} r^2+2r+4 \\ r-2\overline{)r^3 \qquad -8} \\ \underline{r^3-2r^2} \\ 2r^2 \\ \underline{2r^2-4r} \\ 4r-8 \\ \underline{4r-8} \end{array}$$

Since $2^2 - 4 \cdot 4 < 0$, r^2+2r+4 (and hence D^2+2D+4) cannot be factored further

For each of II.–IV. that you decide to do, give a formula for a polynomial differential operator L of least degree which annihilates $f(x)$.

II. (2) $f(x) = 1 + 5x^6$. $D^m(x^k) = 0$ for $k = 0, 1, \ldots, m-1$, so we may choose $\boxed{L = D^7}$. (For $k \geq m$, observe that $D^m(x^k) = k \cdot (k-1) \cdot \ldots \cdot (k-m+1) x^{k-m} \neq 0$, for $x \neq 0$.)

III. (2) $f(x) = x^9 + xe^{7x}$. Since $(D-\alpha)^m(x^k e^{\alpha x}) = 0$ for $k = 0, 1, \ldots, m-1$, and $D^{10}(x^9) = 0$, we take $\boxed{L = D^{10} \cdot (D-7)^2}$.

IV. (2) $f(x) = \cos 4x + e^{2x} \sin 3x$. Since $(D^2 - 2\alpha D + \alpha^2 + \beta^2)(e^{\alpha x} \cos \beta x$ or $e^{\alpha x} \sin \beta x) = 0$, we can take

$$\boxed{L = (D^2+16) \cdot (D^2 - 4D + 13)}.$$

For each of V.–VI. that you decide to do, use the linearity of the the Laplace transform and the formulas derived on Monday to find a formula for the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ of $f(t)$.

V. (2) $f(t) = \frac{1}{8}t^4 + e^{2t}$.

$$\mathcal{L}\{f(t)\} = \frac{1}{8}\mathcal{L}\{t^4\} + \mathcal{L}\{e^{2t}\}$$
$$= \frac{1}{8}\cdot\frac{4!}{s^5} + \frac{1}{s-2}$$
$$= \boxed{\frac{3}{s^5} + \frac{1}{s-2}} \text{ for } s>2.$$

VI. (2) $f(t) = 9 + 2t^3$.

$$\mathcal{L}\{f(t)\} = 9\mathcal{L}\{1\} + 2\mathcal{L}\{t^3\}$$
$$= 9\cdot\frac{1}{s} + 2\cdot\frac{3!}{s^4}$$
$$= \boxed{\frac{9}{s} + \frac{12}{s^4}} \text{ for } s>0.$$

The answers above are obtained from the rules:

$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ for any integer $n \geq 0$ and $s>0$;

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$, $s>a$; and

$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}$ (for well behaved $f(t)$, $g(t)$).