Show all work, and if your solution involves several steps, place a box around your final answer. Question III. is on the back of this page.

I. (2) Let \( R \) be the region in the plane that is enclosed by the lines \( x = 2 \) and \( y = 0 \) and curve \( x = y^{1/3} \) \((= \sqrt[3]{y})\). Let \( f(x, y) \) be a real valued function of two variables which is continuous on \( R \). In each equation below, fill in the four missing limits of integration in the iterated integral shown after the equality symbol in that equation.

\[
\int_0^2 \int_{y^{1/3}}^2 f(x, y) \, dx \, dy
\]

II. (4) Let \( R \) be the region described in I. above. Evaluate the double integral \( \iint_R e^{-x^4} \, dA \). Hint: do not choose the order of integration carefully.

we \( \int_0^2 \int_0^{x^3} e^{-x^4} \, dy \, dx \), since \( \int e^{-x^4} \, dx \) has no closed form solution.

**Sohn. #1:**

\[
\int_0^{x^3} e^{-x^4} \, dy = ye^{-x^4}\bigg|_0^{x^3} = xe^{-x^4} - 0 = xe^{-x^4}
\]

\[
\int_0^2 xe^{-x^4} \, dx = -xe^{-x^4}\bigg|_0^2 = -e^{-16} + 1 = \frac{1}{4} \left(1 - \frac{1}{e^{16}}\right)
\]

\[
\int_0^{x^3} e^{-x^4} \, dx = \int e^{u(-\frac{1}{4})} \, du = -\frac{1}{4}e^u + C = -\frac{1}{4}e^{-x^4} + C
\]

\[
\begin{align*}
\text{let } \, du &= -x^4 \\
\text{let } \, du &= -\frac{4}{3}x \, dx \\
\text{let } \, du &= \frac{3}{4} \, dx
\end{align*}
\]

**Sohn. #2**

\[
-\frac{1}{4} \int_0^{-16} e^u \, du = -\frac{1}{4} e^u\bigg|_0^{-16} = -\frac{1}{4} \left(e^{-16} - 1\right)
\]
III. (4) In one of A. and B., evaluate the given double or iterated integral by converting to polar coordinates.

A. \( \int \int_{R} \sin(\pi(x^2 + y^2)) \, dA \), where \( R \) is the region in the first two quadrants that is bounded by the \( x \)-axis and the upper half of the circle \( x^2 + y^2 = 1 \).

B. \( \int_{0}^{\sqrt{8}} \int_{\sqrt{16 - y^2}}^{\sqrt{16}} 6\sqrt{9 + x^2 + y^2} \, dx \, dy \).

\[
A. \quad \int_{0}^{\pi/2} \int_{0}^{1} \sin(\pi r^2) \, r \, dr \, d\theta = \\
\int_{0}^{\pi/2} \left[ -\frac{\cos(\pi r^2)}{2\pi} \right]_{0}^{1} \, d\theta = \int_{0}^{\pi/2} \left( -\frac{\cos(\pi + 0)}{2\pi} \right) \, d\theta = \frac{\pi}{2} \, d\theta = \frac{\pi}{2} \]

\[
\frac{\pi}{2} \bigg|_{\theta = 0}^{\pi/2} = \frac{\pi}{2}.
\]

\[
B. \quad \int_{0}^{\pi/4} \int_{0}^{4} 6\sqrt{9 + r^2} \, dr \, d\theta = \\
\int_{0}^{\pi/4} \left( 2 \left( 9 + r^2 \right)^{3/2} \right)_{0}^{4} \, d\theta = \\
\int_{0}^{\pi/4} 2 \left( 125 - 27 \right) \, d\theta = \int_{0}^{\pi/4} 196 \, d\theta = 196 \theta \bigg|_{0}^{\pi/4} = \frac{196 \pi}{4} = 49 \pi
\]

Remark: In each of II. and III., each integrand \( f(x,y) \) satisfies \( f(x,y) > 0 \) for all \( (x,y) \in \mathbb{R} \), so each answer provides the volume of the solid \( T = \{(x,y,z) \in \mathbb{R}^3 \mid (x,y) \in R \text{ and } 0 \leq z \leq f(x,y) \} \), which can also be found by evaluating

\[
\iiint_{T} 1 \, dV = \iiint_{R} \left[ \int_{0}^{f(x,y)} 1 \, dz \right] \, dA. \quad \text{E.g., in II.} \quad \iiint_{R} x^3 e^{-x^4} \, dx \, dy \, dz.
\]