I. (1) For each of two of the equations given in the following, name the surface in 3-space defined by that equation.

A. \( x^2 + y^2 - 3z^2 = 1 \) \hspace{5em} \text{hyperboloid of one sheet}

B. \( 4x^2 + y^2 = 9z^2 \) \hspace{5em} \text{elliptic cone}

C. \( 2x^2 + 4y^2 = 3z \) \hspace{5em} \text{elliptic paraboloid}

D. \( x^2 - 3y^2 = z \) \hspace{5em} \text{hyperbolic paraboloid}

II. Let \( L \) be the line defined by \( x = 1 - 2t, y = 5 + 4t, z = 8 + 6t \), and let \( P \) be the plane defined by \( 6x + 4y + z = 29 \).

A. (1) There are infinitely many planes which pass through the point \( Q = (1, 2, 3) \) and are either (i) perpendicular to \( L \) or (ii) parallel to \( L \). Give an equation of a specific example \( M \) of one such plane, and state which of (i) and (ii) is satisfied by your example.

\[
\text{A direction vector for } L \text{ is } \vec{v} = <-2, 4, 6> \text{. If } \vec{n} \text{ is a normal for } M, \text{ then: } \vec{v} \perp \vec{n} \text{ iff (i) holds; and } \vec{v} \parallel \vec{n} \text{ iff (ii) holds. One plane passes through } Q \text{ and satisfies (i):} \quad -2(x-1) + 4(y-2) + 6(z-3) = 0 \quad \text{or} \quad x-2y-3z = -12
\]

Examples passing through \( Q \) and satisfying (i):

\[
\begin{align*}
3(x-1) + (y-3) &= 0; \\
-x-1 + (y-2) - (z-3) &= 0; \\
2(x-1) + (y-2) &= 0; \quad \text{or}...
\end{align*}
\]

B. (3) Determine whether the line \( L \) and plane \( P \) intersect; if so, find the coordinates of the intersection.

Trying to solve
\[
29 = 6(1-2t) + 4(5+4t) + (8+6t)
\]
\[
= 6-12t + 20 + 16t + 8 + 6t
\]
\[
= 34 + 10t \quad \text{or}
\]
\[
-5 = 10t, \text{ we obtain}
\]
\[
t = -\frac{1}{2}
\]

Thus, the point \( (1-2(-\frac{1}{2}), 5+4(-\frac{1}{2}), 8+6(-\frac{1}{2})) = (2, 3, 5) \) is on the line \( L \) and plane \( P \), and it is the only point on \( L \) and \( P \).

\[
\text{\*since } \vec{v} \cdot <3,0,1> = 0 = \vec{v} \cdot <-1,1,-1> = 0 = \vec{v} \cdot <2,1,0>...
\]
III. (5) Let $R(2, 0, 1)$, $S(4, 2, 2)$, and $T(6, 1, 0)$ be points. Do one of A. and B.

A. Find an equation of the plane that passes through the points $R$, $S$, and $T$. Then use your equation to check and show that your plane does contain each of those points.

B. Let $L$ be the line which passes through the points $R$ and $S$. Find the distance between the point $T$ and the line $L$.

A. $\vec{RS} = <2, 2, 1>$ and $\vec{RT} = <4, 1, -1>$. As $<2, 2, 1> \times <4, 1, -1> = <-3, 6, -6>$ we may take as a normal for the plane sought $<-3, 6, -6>$ or, say, $<1, -2, 2>$. An equation for the plane is $x - 6 - 2(y - 1) + 2z = 0$ or $x - 2y + 2z = 4$.

Check:

$R$: $2 - 2 \cdot 0 + 2 \cdot 1 = 4$
$S$: $4 - 2 \cdot 2 + 2 \cdot 2 = 4$
$T$: $6 - 2 \cdot 1 + 2 \cdot 0 = 4$

B. $\frac{|\vec{RS} \times \vec{RT}|}{|\vec{RS}|} = \frac{|<-3, 6, -6>|}{|<2, 2, 1>|} = \frac{\sqrt{9 + 36 + 36}}{\sqrt{4 + 4 + 1}} = \frac{9}{\sqrt{9}} = 3$

Memory aid for B:

$$\frac{d}{|\vec{RT}|} = \sin \theta = \frac{|\vec{RS} \times \vec{RT}|}{|\vec{RS}| \cdot |\vec{RT}|} \Rightarrow$$

$$d = \frac{|\vec{RS} \times \vec{RT}|}{|\vec{RS}|}$$