Show all work, and if your solution involves several steps, place a box around your final answer. Question III. is on the back of this page.

I. (6) Let \( \mathbf{b} = \langle 1, 4, 8 \rangle \) and \( \mathbf{c} = \langle 2, 2, 1 \rangle \). Answer two of A.–C.

A. Find the cosine of the angle \( \theta \) between the vectors \( \mathbf{b} \) and \( \mathbf{c} \).

\[
\cos \theta = \frac{\mathbf{b} \cdot \mathbf{c}}{|\mathbf{b}| |\mathbf{c}|} = \frac{1\cdot 2 + 4\cdot 2 + 8\cdot 1}{\sqrt{1^2 + 4^2 + 8^2} \cdot \sqrt{2^2 + 2^2 + 1^2}} = \frac{18}{9\sqrt{3}} = \frac{2\sqrt{3}}{3}
\]

B. Find the vector projection of \( \mathbf{b} \) onto \( \mathbf{c} \), \( \mathbf{proj}_c \mathbf{b} \).

\[
\mathbf{proj}_c \mathbf{b} = \left( \frac{\mathbf{b} \cdot \mathbf{c}}{\mathbf{c} \cdot \mathbf{c}} \right) \mathbf{c} = \frac{18}{9} \mathbf{c} = 2\mathbf{c} = 2 \langle 2, 2, 1 \rangle = \langle 4, 4, 2 \rangle
\]

C. Find the vector \( \mathbf{b} \times \mathbf{c} \). Using \( \langle b_2 c_3 - b_3 c_2, b_3 c_1 - b_1 c_3, b_1 c_2 - b_2 c_1 \rangle \),

\[
\langle 1, 4, 8 \rangle \times \langle 2, 2, 1 \rangle = \langle 4\cdot 1 - 8\cdot 2, 8\cdot 1 - 1\cdot 4, 1\cdot 2 - 4\cdot 2 \rangle = \langle -12, 15, -6 \rangle.
\]

II. (1) Do one of A. and B, where \( \mathbf{b} \) and \( \mathbf{c} \) are as in question I.

A. Find the vector \( \mathbf{w} = \mathbf{b} - \mathbf{proj}_c \mathbf{b} \), and show that the vectors \( \mathbf{w} \) and \( \mathbf{c} \) are orthogonal to each other.

\[
\mathbf{w} = \langle 1, 4, 8 \rangle - \langle 4, 4, 2 \rangle = \langle -3, 0, 6 \rangle, \text{ and} \]

\[
\mathbf{w} \cdot \mathbf{c} = -3\cdot 0 + 0\cdot 2 + 6\cdot 1 = 0 \Rightarrow \mathbf{w} \perp \mathbf{c}.
\]

B. \( (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{c} = -12, 15, -6 \cdot \langle 1, 4, 8 \rangle = -12 + 60 - 48 = 0, \text{ and} \)

\[
(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{c} = -12, 15, -6 \cdot \langle 2, 2, 1 \rangle = -24 + 30 - 6 = 0 \Rightarrow \mathbf{b} \times \mathbf{c} \perp \mathbf{c} \text{ and } \mathbf{b} \times \mathbf{c} \perp \mathbf{c}.
\]
III. (3) Do one of A. and B.

A. A wagon is pulled horizontally by exerting a constant force of 40 lb on the handle at an angle of \( \pi/3 \) with the horizontal. How much work is done in moving the wagon 15 ft?

One may take the force \( \vec{F} \) to be \( \vec{F} = 40 <\cos(\pi/3), \sin(\pi/3)> = <20, 20\sqrt{3}> \) and the displacement \( \vec{D} \) to be \( \vec{D} = (0,0)(15,0) = <15,0>. \) Then the work \( W \) done is \( W = \vec{F} \cdot \vec{D} = 20 \cdot 15 + 20\sqrt{3} \cdot 0 = 300 \text{ ft-lb} \) (since \( W = (\text{scalar projection of } \vec{F} \text{ onto } \vec{D}) \cdot |\vec{D}| = \frac{\vec{F} \cdot \vec{D}}{|\vec{D}|} \cdot |\vec{D}| = \vec{F} \cdot \vec{D} \).

B. Find a set of parametric equations for the line \( L \) which passes through the points (2, 4, 6) and (12, 10, 8).

A direction vector for \( L \) is \( \vec{P}_0 \vec{P}_1 = <12-2, 10-4, 8-6> = <10, 6, 2>. \) A set of parametric equations for \( L \) is

\[
\begin{align*}
x &= 2 + 10t \\
y &= 4 + 6t \\
z &= 6 + 2t
\end{align*}
\]

or

\[
\begin{align*}
x &= 12 + 10t \\
y &= 10 + 6t \\
z &= 8 + 2t
\end{align*}
\]

or

\[
\begin{align*}
x &= 2 + 5t \\
y &= 4 + 3t \\
z &= 6 + 2t
\end{align*}
\]

or

\[
\begin{align*}
x &= 2 - 5t \\
y &= 4 - 3t \\
z &= 6 - 2t
\end{align*}
\]

Note: for any specific numbers \( x_0, y_0, z_0, a, b, c \),

\[
\begin{align*}
x &= x_0 + at \\
y &= y_0 + bt \\
z &= z_0 + ct
\end{align*}
\]

is also a set of parametric equations for \( L \), provided that \((x_0, y_0, z_0)\) is a point on \( L \) and \(<a, b, c> = \vec{P}_0 \vec{P}_1 \neq 0 \) for some constant \( k \neq 0 \).