**Theorem.** Suppose that $\lim_{n \to \infty} a_n = a$, then prove that $\lim_{n \to \infty} |a_n| = |a|$. 

**Theorem.** Convergent sequences are bounded.

**Defn.** A sequence $\{a_n\}$ is called **monotone increasing** if $a_m \leq a_n$ whenever $m \leq n$. A sequence $\{a_n\}$ is called **monotone decreasing** if $a_n \leq a_m$ whenever $m \leq n$.

**Theorem.** Monotone sequences, which are also bounded, converge.

**Theorem.** Suppose that $\lim_{n \to \infty} a_n = a$ and $\lim_{n \to \infty} b_n = a$. If $a_n \leq c_n \leq b_n$ for all $n \in \mathbb{N}$, then $\lim_{n \to \infty} c_n$ exists and equals $a$.

**Theorem.** (Properties of Limits) Suppose that $\lim_{n \to \infty} a_n = a$ and $\lim_{n \to \infty} b_n = b$, then

1. $\lim_{n \to \infty} a_n + b_n = a + b$

2. $\lim_{n \to \infty} a_nb_n = ab$

3. If $b \neq 0$, then $\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{a}{b}$.

**Defn.** A sequence $\{a_n\}$ is called **Cauchy** if for each $\epsilon > 0$ there is an $N \in \mathbb{N}$ so that $|a_m - a_n| < \epsilon$ whenever $m, n \geq N$.

**Theorem.** Each convergent sequence is Cauchy.

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