Name: __________________________

Directions: Answers should be provided in complete sentences with justifications.

1. Suppose \( f : A \rightarrow B \) where \( (A, d_A), (B, d_B) \) are metric spaces.
   a) Give the definition for a function \( f \) to be continuous at a point \( x_0 \in A \).
   
   b) If \( x_0 \) is an isolated point of \( A \), using the definition, prove that every function \( f : A \rightarrow B \) is continuous at \( x_0 \).

2. Suppose \( f : A \rightarrow B \) is a continuous function at a point \( x_0 \), then prove that whenever a sequence \( \lbrace x_n \rbrace_{n=1}^{\infty} \) is a sequence which converges to \( x_0 \), then the corresponding \( y_n := f(x_n) \) form a sequence in \( B \) which converges to the point \( y_0 := f(x_0) \).
3. Prove that the composition of two continuous functions is continuous.
   *(Note: You may prove this using any of the four characterizations of continuity that we had in our theorem, i.e. sequences, inverse images of open sets, . . .)*

4. Consider a metric space \( A \) and a subset \( C \subseteq A \).
   a) Define a *disconnection* for a subset \( C \).
   
   b) Define what it means for a set \( C \) to be *connected*.
   
   c) Give an example of a subset of the real numbers which is connected and one which is not.
   
   d) What precisely are the connected subsets of the real numbers?

5. Let \( R(x) \) be any rational function on the real numbers (i.e. \( R \) is a quotient of polynomials).
   a) What is the domain of \( R \) and at which points \( x \) is \( R \) continuous?
   
   b) What is the domain of \( f(x) := \sqrt{R(x)} \) and where is it continuous?

7. a) Define what it means for a set $K \subseteq A$ to be a compact set.

b) Give an example of a subset of the real numbers which is compact and one which is not.

8. State the Heine-Borel Theorem.