1. [Warmup] Give an example of each of the following for the metric space of real numbers (you do not need to justify).

(a) an open set which is not an open interval.

(b) an infinite closed set which is not a closed interval.

(c) a set which is closed, but has no limit points.

(d) a set which is open, but has no limit points.

(e) a sequence which is bounded, but is not convergent.

(f) a sequence which is convergent, but is not monotone.
2. Using the definition of “convergence of a sequence,” prove that if \( \{b_n\} \) converges to \( b \) \((b \neq 0)\), then \( \left\{ \frac{1}{b_n} \right\} \) converges to \( \frac{1}{b} \).

3. Using the properties of limits, determine whether or not the following limit exists. Be sure to state which property you are using as you show your work.

\[
\lim_{n \to \infty} \frac{1 + \sqrt{n}}{3 - n}.
\]
4. a.) Give the definition of an **open \( \epsilon \)-neighborhood** of a real number \( x_0 \).

b.) Give the definition of an **open** set of real numbers.

c.) Prove that intersection of a finite number of open sets is open.

5. a.) Define **limit point** for a set \( C \) of real numbers.

b.) Define “limit of a function at a point \( x_0 \).”

c.) Using the definition, prove that \( \lim_{x \to \frac{1}{4}} \sqrt{x} = \frac{1}{2} \).
6. a.) Give the definition for a function $f$ to be continuous at a point $x_0$.

b.) State an equivalent condition (involving sequences) in order to verify that a function is continuous at $x_0$.

c.) Using properties of limits and part b), show that $f(x) = \frac{x^2 + 1}{\sqrt{x} + 2}$ is continuous at $x_0 = 2$.

7. Negate the statement that a function is continuous at a point.