Name: ____________________________

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. Pick exactly two of the following three parts to work: Suppose that $F$ is an ordered field,

(a) prove for each $a \in F$, $a \cdot 0 = 0$.

(b) prove that $0 < 1$.

(c) if $a < b$ and $c < d$, prove that $a + c < b + d$. 
2. (a) Give a precise definition for a set to be finite.

(b) Give a precise definition for a set to be countably infinite.

3. Let $A$ be a nonempty subset of $\mathbb{R}$.

a.) Define ‘upper bound’ for $A$.

b.) Define ‘least upper bound’ for $A$.

c.) Prove that least upper bounds are unique.
4. a. State and prove the Archimedean principle.

b. Prove that for each $\epsilon > 0$, there exists a natural number $N$ such that for all $N \leq n$ there holds $0 < \frac{1}{n} < \epsilon$.

5. Negate the statement:

for each $\epsilon > 0$ there is a natural number $N$ such that for every $n \geq N$ it is implied that $|a_n - a| < \epsilon$
6. For $a > 0$, and all natural numbers $n$, prove that
\[ 1 + na < (1 + a)^n \]

7. Prove that if $0 < r < 1$ and $\epsilon > 0$, then there exists a natural number $n$ so that $r^n < \epsilon$. (Hint: Problem #7)