Theorem. Suppose that \( \lim_{n \to \infty} a_n = a \), then prove that \( \lim_{n \to \infty} |a_n| = |a| \).

Theorem. Convergent sequences are bounded.

Defn. A sequence \( \{a_n\} \) is called *monotone increasing* if \( a_m \leq a_n \) whenever \( m \leq n \). A sequence \( \{a_n\} \) is called *monotone decreasing* if \( a_n \leq a_m \) whenever \( m \leq n \).

Theorem. Monotone sequences, which are also bounded, converge.

Theorem. Suppose that \( \lim_{n \to \infty} a_n = a \) and \( \lim_{n \to \infty} b_n = a \). If \( a_n \leq c_n \leq b_n \) for all \( n \in \mathbb{N} \), then \( \lim_{n \to \infty} c_n \) exists and equals \( a \).

Theorem. (Properties of Limits) Suppose that \( \lim_{n \to \infty} a_n = a \) and \( \lim_{n \to \infty} b_n = b \), then

1. \( \lim_{n \to \infty} a_n + b_n = a + b \)

2. \( \lim_{n \to \infty} a_n b_n = ab \)

3. If \( b \neq 0 \), then \( \lim_{n \to \infty} \frac{a_n}{b_n} = \frac{a}{b} \).

Defn. A sequence \( \{a_n\} \) is called *Cauchy* if for each \( \epsilon > 0 \) there is an \( N \in \mathbb{N} \) so that \( |a_m - a_n| < \epsilon \) whenever \( m, n \geq N \).

Theorem. Each convergent sequence is Cauchy.

Theorem. Each Cauchy sequence is convergent.