Complex Variables
(Math 552 – 752I)
Test 3 – November 30, 2000

1	(20 pts)
2	(16 pts)
3	(16 pts)
4	(16 pts)
5	(16 pts)
6	(16 pts)
7	(16 pts)

Name: ____________________________

Directions: Show your work for full credit. Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room.

1. State Cauchy's theorem and sketch its proof.

Work any 5 of the following 6 problems. Be sure to indicate which 5 you wish to be graded.

2. Compute the line integral $\int_\Gamma -x\,dx + y\,dy$, where Γ is the directed circular line segment from $(1,0)$ to $(0,1)$

3. a.) Parameterize the region Ω which is the interior of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,1)$.
   b.) Compute $\oint_\Gamma \bar{z}\,dz$ where Γ is the boundary of Ω traversed once in the positive direction.

4. Use Green's theorem to compute the line integral $\oint_\Gamma (-y)\,dx + x\,dy$, where Γ is the perimeter of the upper unit semicircle with center $(0,0)$ traversed once in the counterclockwise direction.

5. Use partial fractions to compute $\int_\Gamma \frac{z}{z^2+1}\,dz$ where Γ is the positively oriented circle about i of radius 1.

6. Compute
$$\oint_\Gamma \frac{\sin(z)}{z^2+1}dz$$
where Γ is the curve parameterized as $z(t) = 2e^{it} + 1,\ 0 \le t \le 2\pi$. Describe this curve.

7. Compute
$$\oint_\Gamma \frac{\cos(z)}{(2z-\pi)^3}dz$$
where Γ is the counterclockwise circle of radius 2 and center the origin.

Extra Credit (15 pts.)

Show that if f is analytic on a region containing the simple, closed, piecewise-smooth curve Γ, and z_0 does not lie on Γ, then
$$\oint_\Gamma \frac{f'(z)}{z-z_0}\,dz = \oint_\Gamma \frac{f(z)}{(z-z_0)^2}\,dz$$