Complex Variables
(Math 552 – 752I)
Test 2 – October 26, 2000

Name: ____________________________

Directions: Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room.

1	(10 *pts*)
2	(15 *pts*)
3	(10 *pts*)
4	(15 *pts*)
5	(15 *pts*)
6	(17 *pts*)
7	(18 *pts*)

1. Show that $\lim\limits_{z\to 0} \dfrac{\cos(2z)-1}{z} = 0$. (Hint: $f'(z_0) = \lim_{z\to 0} \frac{f(z_0+z)-f(z_0)}{z}$)

2. a.) Verify that the function $u(x,y) = 2y - \exp(y)\sin(x) + 1$ is harmonic for all complex $z = x + iy$.
   b.) Compute all harmonic conjugates v of u.

3. a.) Define $\sin(z)$.
   b.) Define $\sinh(z)$.
   c.) Verify the identity: $\sinh(z) = -i\sin(iz)$.

4. Compute each of the following and write in the form $a + ib$:

   a.) all values of $\log(i)$.
   b.) $\cos(\pi - 2i)$
   c.) $\mathrm{Log}(i\mathrm{e})$

5. Solve for all z for which
   a.) $e^z = 2 - 2i$
   b.) $\sinh(z) = i$.

6. Using the definition of β^α and selecting the principal branch, set $f(z) := i^z$ (z complex).
   a.) Determine the natural domain of f.
   b.) Compute all values of i^{1-i}. What is its principal value, i.e. $f(1-i)$?
   c.) Prove that $f'(z) = \dfrac{i\pi}{2}\, i^z$.

7. a.) Parameterize the circle γ with radius 3 and center $-2+3i$ which is traversed once in the counterclockwise direction.
   b.) Parameterize the straight line segment γ from $z = -1+2i$ to $z = 1+i$ and compute the path integral $\displaystyle\int_\gamma 2x - y\, ds$.

Extra Credit Suppose that f is an entire function such that $\mathrm{Imag}(f)$ is constant, then prove that f is constant.