Complex Variables
(Math 552 – 752I)
Test 1 – September 21, 2000

1		(14*pts*)
2		(14*pts*)
3		(15*pts*)
4		(16*pts*)
5		(10*pts*)
6		(16*pts*)
7		(15*pts*)

Name: ______________________________

Directions: Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room.

1. Simplify the following expressions into the form of $a + ib$ with a, b real numbers:
   a.) $\dfrac{i(1-i)^2}{1+i}$
   b.) $(i+1)^{15}$

2. Sketch each of the following subsets of the complex numbers. For each one, determine the set's interior points and limit points. Determine if the set is open, closed, or a domain.
   a.) $\{z \in C \mid 1 \leq |z - 2i| < 3\}$
   b.) $\{z \in C \mid \text{Real}(z) > 0\} \cup \{z \in C \mid |z| < 1\}$

3. Compute all fourth roots of $-8\sqrt{3} + 8i$.

4. a.) Give the definition of the limit of a function at a point z_0: $\lim_{z \to z_0} f(z) = L$.
   b.) Determine $\lim_{n \to \infty} z_n$, if $z_n = \dfrac{n(3+i)}{n+1}$.

5. Using the definition of the derivative, show that $f'(z) = 2z$ if $f(z) = z^2$.

6. Using the Cauchy Riemann conditions, determine which of the following functions are analytic. For those functions that are analytic, compute $f'(z)$. (Note: $z = x + iy$).
   (a) $(x^3 - 3xy^2) + i\ (3x^2y - y^3)$
   (b) $e^{-x}\cos(y) + i\ e^{x}\sin(y)$

7. Determine the domain of each function and at which points it is analytic.
   (a) $f(z) = e^{-2z}$
   (b) $g(z) = \dfrac{2z - 3}{1 + z^2}$