Complex Variables
(Math 552 – 752I)
Final Exam – December 14, 2000

Name: ____________________________

Directions: Show your work for full credit. Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room.

1	(10 pts)
2	(10 pts)
3	(15 pts)
4	(10 pts)
5	(10 pts)
6	(10 pts)
7	(10 pts)
8	(15 pts)
9	(15 pts)

1. Simplify the following expressions into the form of $a + ib$ with a, b real numbers:
   a. $\dfrac{(2i+1)(i-1)}{i+1}$
   b. all cube roots of $\sqrt{12} - 2i$ *(Leave this one in polar form)*
   c. the principal value of $(1-i)^i$

2. a. State the Cauchy-Riemann equations.
   b. Use them to determine if
   $$f(z) := \frac{\cos(y) - i\sin(y)}{\exp(x)}$$
   is entire, where $z = x + iy$.

3. State the Fundamental Theorem of Algebra.

4. State Cauchy's theorem and sketch its proof.

5. Compute directly (i.e. without using Green's theorem) the line integral $\oint_\Gamma (-y)\ dx + x\ dy$ around the upper semi-circle with radius one and center the origin.

6. Use Green's theorem to compute the line integral $\oint_\Gamma (-x^2)\ dx + (\exp(y) + 5x)\ dy$, where Γ is the perimeter of the unit circle with center $(0,0)$ traversed once in the counterclockwise direction.

7. Compute
   $$\oint_\Gamma \frac{\sin(z)}{z^2 - z}\ dz$$
   where Γ is the curve parameterized by $z(t) := 2e^{it} + 1,\ 0 \le t \le 2\pi$.

8. State Liouville's theorem and sketch its proof.

9. Compute

$$\oint_{|z|=1} \frac{\exp(k\, z^n)}{z}\, dz$$

and use the result to show that

$$\int_0^{2\pi} \exp(k\cos(n\theta))\cos(k\sin(n\theta))d\theta = 2\pi.$$

(*Hint: Consider the imaginary part.*)