PROBABILITY
(MATH/STAT 511)
TEST 3 – APRIL 4, 2001

Directions: Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room. Calculators are allowed, but you must show intermediate work for partial credit.

1. Flaws occur on an average of 5 per every 10 meters of computer tape. If these are distributed as a Poisson process, then

   a.) give the probability of exactly 15 flaws in 20 meters of tape. (You may leave your answers in unsimplified form.)
   b.) what is the probability that no flaws appear in the first 20 meters of tape? (You may leave your answers in unsimplified form.)

2. The probability that a machine produces a defective item is 0.10. Each item is checked as it is produced. Assume that these are independent trials and compute the probability that at least 10 items must be checked to find one that is defective.

3. Compute the following for the continuous random variable X with probability mass function

$$f(x) := \begin{cases} 2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) cumulative distribution,

   (b) mean

   (c) variance.

4. Show that the moment generating function $M(t)$ for the **standard normal** distribution is $e^{\frac{1}{2}t^2}$.

5. Telephone calls arrive at a support services center according to a Poisson process on the average of two every five minutes. Let X denote the waiting time until the first call that arrives after 9 a.m.
   a.) What is the p.d.f. of X and the corresponding required parameters?
   b.) Compute the probability that X is greater than 6 minutes.

6. Suppose Z is distributed as a standard normal random variable, then compute
   a) $P(Z \leq \frac{1}{2})$
   b) $P(|Z| \leq \frac{1}{2})$
   c) If $Y = u(Z)$ is a random variable defined by $u(z) = 1 + 2z$, then show that $E[Y] = 1$.
   d) For this same Y, compute $E[Y^2]$.