Probability (Math/Stat 511) Test 2 – March 9, 2001

Name: _____

- **Directions:** Answer all questions in the space provided. You can also use the back of the facing opposite page if you need more room. Calculators are allowed, but you must show intermediate work for partial credit.
 - 1. Define each of the following terms:

a.) hypergeometric probability distribution. Describe the typical random variable X which has this as its probability mass function.

- b.) the *expectation* of a random variable, as given in class.
- c.) variance of a random variable X.
- d.) the moment generating function for a random variable X.

2. Prove that
$$\sigma^2 = E[X^2] - E[X]^2$$
.

- 3. Suppose the probability that a basketball player makes a free throw is 70% and that each attempt is independent of the rest.
 - a.) What is the probability that on their fourth attempt, the player makes a free throw?
 - b.) If a player has eight attempts, what is the probability that they do not make them all?
 - c.) If the random variable X is defined as the number of free throws made in the eight attempts,
 - what is the **type** of the probability distribution function of X?
 - d.) What are the mean and variance of X?
- 4. If X is a Binomial distribution with mean $\mu = 12$ and $\sigma^2 = 3$, then compute $P(1 \le X \le 4)$.
- 5. Let X be a discrete random variable with probability mass function

$$f(x) = \frac{4 - |5 - x|}{16}, \quad x = 2, 3, 4, 5, 6, 7, 8$$

- a.) Compute the mean and variance of X.
- b.) Compute the moment generating function for X.
- 6. If the moment generating function of a random variable X is given by

$$M(t) = \frac{1}{12} \left(1 + 2e^{2t} \right) \left(1 + 3e^{-t} \right),$$

then compute the

- a.) probability mass distribution of X,
- b.) expectation of X.
- 7. If the moment generating function of a random variable X is given by

$$M(t) = (.8 + .2e^t)^4,$$

then compute the variance of X.

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