Differential Equations
(Math 242.01)
Practice Final Exam

1. Find the solution (in implicit form if necessary) of each of the following differential equations:

   (a) $y' = ye^{2x} + 2e^{4x}, \quad y(1) = 3$
   (b) $2\frac{dy}{dx} = \frac{x}{y}(x^2 - 16)^{-1/2}, \quad y(5) = 4$

2. A 200 gallon tank initially contains 50 pounds of salt in a mixture of 100 gallons of brine. Another brine containing 1 lb. of salt per gallon is being added to the tank at a rate of 5 gallons per minute. The mixed brine in the tank flows out at a rate of 2 gallons per minute. When the tank is full, how many pounds of salt per gallon will the resulting brine solution contain?

3. Find the general solution (in implicit form if necessary) of each of the following differential equations:

   (a) $y' = \frac{x-y}{x+y}$
   (b) $y''' + y'' - 2y' = 0$
   (c) $y''' - 4y'' + 4y' = 0$
   (d) $y'' + 6y' + 10 = 0$

4. Determine the solution of each of the following initial value problems:

   (a) $y''' + y'' - 2y' = 0, \quad y(0) = 9,\ y'(0) = 0,\ y''(0) = 12$
   (b) $y'' - y = 2x\ y(0) = 1,\ y'(0) = 1$

5. Determine the general solution of each of the following:

   (a) $y''' - 4y'' = cos(x)$
   (b) $y'' + y = e^x + 2$

6. Use variation of parameters to find a particular solution for the differential equation $y'' - 4y' = 2e^{3t}$.

7. Compute the Laplace transform of each of the following:

   a.) $f(t) := \begin{cases} 0, & 0 < t < 2; \\ 1, & 2 \le t < 4. \\ 0, & 4 \le t < 4. \end{cases}$ by using the unit step function.

   b.) $te^{3t}\sin(2t)$

8. Compute the inverse Laplace transform of each of the following:

   a.) $\dfrac{s}{s^2 - 10s + 29}$

   b.) $\dfrac{e^{-3s}}{s^2 + 1}$

9. Using the Laplace transform, compute the solution to the following equation:
$y'' - 3y' + 2y = (t+1)^2, \quad y(0) = -1, y'(0) = 2.$

10. Use step size $h = .1$ and apply each of the following numerical methods

a) Euler

b) Improved Euler

to estimate $y(.2)$ where y is the solution to the equation $y' = x^2 - y^2, \quad y(0) = 1.$