MATH 142 - CALCULUS II (SECTIONS 11-12)
SAMPLE PROBLEMS FOR TEST 3 – APRIL 7, 2005

Name: ______________________________

Directions: Calculators will be allowed this Test. To receive proper credit however, you must show your intermediate work and *box* your final answer.

1. Compute the first 6 terms of the Taylor series for each of the following functions:

   (a) $\sqrt{1+x}$

   (b) $\exp(x^2)$

   (c) $\ln(1+x)$

   (d) $\cosh(x)$

   (e) $\exp(6x-1)$

2. Compute the following integrals, stating (i) whether they are proper or improper, and (ii) whether or not they exist. If improper, state why they are so, if they exist, determine their value.

   (a) $\int_0^1 x^{\frac{1}{2}}\, dx$

   (b) $\int_0^1 x^{-\frac{1}{2}}\, dx$

   (c) $\int_0^1 x^{-2}\, dx$

   (d) $\int_1^\infty x^{-2}\, dx$

   (e) $\int_1^\infty x \exp(-x^2)\, dx$

   (f) $\int_0^2 \frac{1}{x-1}\, dx$

3. Compute the following numerical approximations to $\int_{-1}^2 \frac{1}{x^2+1}\, dx$

   (a) trapezoidal rule with $n = 6$ total intervals

   (b) Simpson's rule with $2n = 6$ total intervals

4. For each of the following sequences determine whether it is (i) either eventually increasing or decreasing, (ii) bounded, (iii) convergent. Justify your answers.

   (a) $\left\{\dfrac{n}{1+n^2}\right\}_{n=1}^{\infty}$

   (b) $\left\{\dfrac{n!}{n^n}\right\}_{n=1}^{\infty}$

   (c) $\left\{\dfrac{exp(2n+1)}{n!}\right\}_{n=1}^{\infty}$

(d) $\left\{ \dfrac{n}{\sqrt{2n^2+1}} \right\}_{n=1}^{\infty}$

5. Determine whether or not each of the following series converges. If it converges, then determine its limit.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{e^k}$

   (b) Express $.2315315315\ldots$ first as a geometric series, then as a rational number.

   (c) $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{2}{3}\right)^{k+1}$

   (d) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{3k^2+2}}$