Math 142 - Calculus II (Sections 11-12)
Final Exam – April 30, 2005

1	(5 *pts*)
2	(10 *pts*)
3	(5 *pts*)
4	(35 *pts*)
5	(5 *pts*)
6	(5 *pts*)
7	(5 *pts*)
8	(5 *pts*)
9	(15 *pts*)
10	(10 *pts*)

Name: ____________________________

Directions: Calculators are allowed on this Exam. To receive proper credit however, you must show your intermediate work and *box* your final answer.

1. Compute the area bounded between the curves $y = x^3$ and $y = 4x$.
2. Compute the volume of revolution of each:
   (a) about the x-axis of the region bounded by the curves $x = \sqrt{y}$ and $x = y/4$.
   (b) about the y-axis of the region bounded by the curves $y = x^2$, $y = 0$ and $x = 1$.
3. Differentiate $y = \exp(x \ln(x^2 + 1))$.
4. Compute each of the following integrals:
   (a) $\int x^2 \arctan(2x)\, dx$
   (b) $\int \sin(\frac{1}{2}x) \cos(2x)\, dx$
   (c) $\displaystyle\int \frac{1}{(1 - 9x^2)^{\frac{3}{2}}}\, dx$
   (d) $\displaystyle\int \frac{x}{(x^2 + 1)}\, dx$
   (e) $\displaystyle\int \frac{2x^2 + 3}{x(x - 1)^2}\, dx$
   (f) $\displaystyle\int \frac{2x^2 + 3}{x(x^2 - 1)}\, dx$
   (g) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{1 - x}}\, dx$

5. Use Simpson's rule to estimate the integral $\int_1^3 (2+x)\, dx$ with $2n = 6$ subintervals. Sketch a graph of what you are doing.

6. Determine if the sequence $\left\{ \dfrac{n}{3n-1} \right\}_{n=1}^{\infty}$ converges. Justify your answer and compute the limit if it exists.

7. Compute the Taylor polynomial of degree 5 of the function $f(x) = e^{2x+1}$ about $x_0 = 0$.

8. Determine for what values of a the series $\sum_{k=1}^{\infty} (2/a)^k$ converges.

9. Determine if each of the series converges by stating the 'Test' you are using and verify the criteria. No credit can be given unless a proper test is applied:

   (a) $\sum_{k=1}^{\infty} \dfrac{1}{20k+100}$

   (b) $\sum_{k=1}^{\infty} (-1)^k \dfrac{256+e^3}{\sqrt{k}}$

   (c) $\sum_{k=1}^{\infty} \dfrac{k}{2k+1}$

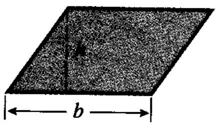
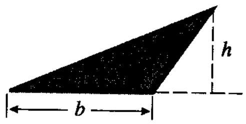
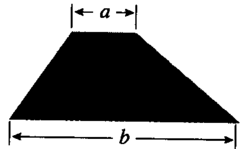
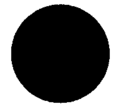
10. Determine the interval of convergence for each of the following series:

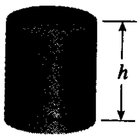
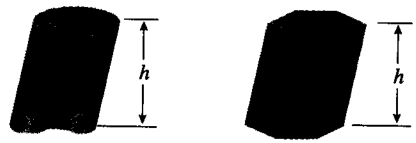
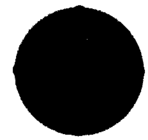
    (a) $\sum_{k=1}^{\infty} \dfrac{(-1)^k (2x)^k}{k!}$

    (b) $\sum_{k=1}^{\infty} \dfrac{(-1)^k}{3^k k^2} x^k$

GEOMETRY FORMULAS

A = area, S = lateral surface area, V = volume, h = height, B = area of base, r = radius, l = slant height, C = circumference, s = arc length

Parallelogram	Triangle	Trapezoid	Circle	Sector
				
$A = bh$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}(a+b)h$	$A = \pi r^2$, $C = 2\pi r$	$A = \frac{1}{2}r^2\theta$, $s = r\theta$ (θ in radians)

Right Circular Cylinder	Right Circular Cone	Any Cylinder or Prism with Parallel Bases	Sphere
			
$V = \pi r^2 h$, $S = 2\pi rh$	$V = \frac{1}{3}\pi r^2 h$, $S = \pi rl$	$V = Bh$	$V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$

ALGEBRA FORMULAS

THE QUADRATIC FORMULA	THE BINOMIAL FORMULA
The solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$(x+y)^n = x^n + nx^{n-1}y + \dfrac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \dfrac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots + nxy^{n-1} + y^n$ $(x-y)^n = x^n - nx^{n-1}y + \dfrac{n(n-1)}{1\cdot 2}x^{n-2}y^2 - \dfrac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots \pm nxy^{n-1} \mp y^n$

TABLE OF INTEGRALS

BASIC FUNCTIONS

1. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C$
2. $\displaystyle\int \frac{du}{u} = \ln|u| + C$
3. $\displaystyle\int e^u\,du = e^u + C$
4. $\displaystyle\int \sin u\,du = -\cos u + C$
5. $\displaystyle\int \cos u\,du = \sin u + C$
6. $\displaystyle\int \tan u\,du = \ln|\sec u| + C$
7. $\displaystyle\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$
8. $\displaystyle\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$
9. $\displaystyle\int \tan^{-1} u\,du = u\tan^{-1} u - \ln\sqrt{1+u^2} + C$
10. $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + C$
11. $\displaystyle\int \ln u\,du = u\ln u - u + C$
12. $\displaystyle\int \cot u\,du = \ln|\sin u| + C$
13. $\displaystyle\int \sec u\,du = \ln|\sec u + \tan u| + C = \ln\left|\tan\left(\tfrac{1}{4}\pi + \tfrac{1}{2}u\right)\right| + C$
14. $\displaystyle\int \csc u\,du = \ln|\csc u - \cot u| + C = \ln\left|\tan \tfrac{1}{2}u\right| + C$
15. $\displaystyle\int \cot^{-1} u\,du = u\cot^{-1} u + \ln\sqrt{1+u^2} + C$
16. $\displaystyle\int \sec^{-1} u\,du = u\sec^{-1} u - \ln\left|u + \sqrt{u^2-1}\right| + C$
17. $\displaystyle\int \csc^{-1} u\,du = u\csc^{-1} u + \ln\left|u + \sqrt{u^2-1}\right| + C$

RECIPROCALS OF BASIC FUNCTIONS

18. $\int \frac{1}{1 \pm \sin u}\, du = \tan u \mp \sec u + C$

19. $\int \frac{1}{1 \pm \cos u}\, du = -\cot u \pm \csc u + C$

20. $\int \frac{1}{1 \pm \tan u}\, du = \frac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

21. $\int \frac{1}{\sin u \cos u}\, du = \ln|\tan u| + C$

22. $\int \frac{1}{1 \pm \cot u}\, du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

23. $\int \frac{1}{1 \pm \sec u}\, du = u + \cot u \mp \csc u + C$

24. $\int \frac{1}{1 \pm \csc u}\, du = u - \tan u \pm \sec u + C$

25. $\int \frac{1}{1 \pm e^u}\, du = u - \ln(1 \pm e^u) + C$

POWERS OF TRIGONOMETRIC FUNCTIONS

26. $\int \sin^2 u\, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

27. $\int \cos^2 u\, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

28. $\int \tan^2 u\, du = \tan u - u + C$

29. $\int \sin^n u\, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\, du$

30. $\int \cos^n u\, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\, du$

31. $\int \tan^n u\, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du$

32. $\int \cot^2 u\, du = -\cot u - u + C$

33. $\int \sec^2 u\, du = \tan u + C$

34. $\int \csc^2 u\, du = -\cot u + C$

35. $\int \cot^n u\, du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\, du$

36. $\int \sec^n u\, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du$

37. $\int \csc^n u\, du = -\frac{1}{n-1}\csc^{n-2} u \cot u + \frac{n-2}{n-1}\int \csc^{n-2} u\, du$

PRODUCTS OF TRIGONOMETRIC FUNCTIONS

38. $\int \sin mu \sin nu\, du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

39. $\int \cos mu \cos nu\, du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

40. $\int \sin mu \cos nu\, du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

41. $\int \sin^m u \cos^n u\, du = -\frac{\sin^{m-1} u \cos^{n+1} u}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} u \cos^n u\, du$

$$= \frac{\sin^{m+1} u \cos^{n-1} u}{m+n} + \frac{n-1}{m+n}\int \sin^m u \cos^{n-2} u\, du$$

PRODUCTS OF TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS

42. $\int e^{au} \sin bu\, du = \frac{e^{au}}{a^2+b^2}(a \sin bu - b \cos bu) + C$

43. $\int e^{au} \cos bu\, du = \frac{e^{au}}{a^2+b^2}(a \cos bu + b \sin bu) + C$

POWERS OF u MULTIPLYING OR DIVIDING BASIC FUNCTIONS

44. $\int u \sin u\, du = \sin u - u \cos u + C$

45. $\int u \cos u\, du = \cos u + u \sin u + C$

46. $\int u^2 \sin u\, du = 2u \sin u + (2 - u^2)\cos u + C$

47. $\int u^2 \cos u\, du = 2u \cos u + (u^2 - 2)\sin u + C$

48. $\int u^n \sin u\, du = -u^n \cos u + n\int u^{n-1} \cos u\, du$

49. $\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1} \sin u\, du$

50. $\int u^n \ln u\, du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

51. $\int ue^u\, du = e^u(u-1) + C$

52. $\int u^n e^u\, du = u^n e^u - n\int u^{n-1} e^u\, du$

53. $\int u^n a^u\, du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a}\int u^{n-1} a^u\, du + C$

54. $\int \frac{e^u\, du}{u^n} = -\frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^u\, du}{u^{n-1}}$

55. $\int \frac{a^u\, du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1}\int \frac{a^u\, du}{u^{n-1}}$

56. $\int \frac{du}{u \ln u} = \ln|\ln u| + C$

POLYNOMIALS MULTIPLYING BASIC FUNCTIONS

57. $\int p(u)e^{au}\, du = \frac{1}{a}p(u)e^{au} - \frac{1}{a^2}p'(u)e^{au} + \frac{1}{a^3}p''(u)e^{au} - \cdots$ [signs alternate: $+ - + - \cdots$]

58. $\int p(u) \sin au\, du = -\frac{1}{a}p(u)\cos au + \frac{1}{a^2}p'(u)\sin au + \frac{1}{a^3}p''(u)\cos au - \cdots$ [signs alternate in pairs after first term: $+ + - - + + - - \cdots$]

59. $\int p(u) \cos au\, du = \frac{1}{a}p(u)\sin au + \frac{1}{a^2}p'(u)\cos au - \frac{1}{a^3}p''(u)\sin au - \cdots$ [signs alternate in pairs: $+ + - - + + - - \cdots$]

RATIONAL FUNCTIONS CONTAINING POWERS OF $a + bu$ IN THE DENOMINATOR

60. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}[bu - a\ln|a+bu|] + C$

61. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln|a+bu|\right] + C$

62. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

63. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

64. $\displaystyle\int \frac{u\,du}{(a+bu)^3} = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

65. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

66. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

67. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

RATIONAL FUNCTIONS CONTAINING $a^2 \pm u^2$ IN THE DENOMINATOR ($a > 0$)

68. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

69. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

70. $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

71. $\displaystyle\int \frac{bu+c}{a^2+u^2}\,du = \frac{b}{2}\ln(a^2+u^2) + \frac{c}{a}\tan^{-1}\frac{u}{a} + C$

INTEGRALS OF $\sqrt{a^2+u^2}$, $\sqrt{a^2-u^2}$, $\sqrt{u^2-a^2}$ AND THEIR RECIPROCALS ($a > 0$)

72. $\displaystyle\int \sqrt{u^2+a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

73. $\displaystyle\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

74. $\displaystyle\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

75. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$

76. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u+\sqrt{u^2-a^2}| + C$

77. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a^2-u^2}$ OR ITS RECIPROCAL

78. $\displaystyle\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$

79. $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

80. $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u^2} = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1}\frac{u}{a} + C$

81. $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

82. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

83. $\displaystyle\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{u^2 \pm a^2}$ OR THEIR RECIPROCALS

84. $\displaystyle\int u\sqrt{u^2+a^2}\,du = \frac{1}{3}(u^2+a^2)^{3/2} + C$

85. $\displaystyle\int u\sqrt{u^2-a^2}\,du = \frac{1}{3}(u^2-a^2)^{3/2} + C$

86. $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

87. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

88. $\displaystyle\int \frac{\sqrt{u^2-a^2}\,du}{u} = \sqrt{u^2-a^2} - a\sec^{-1}\left|\frac{u}{a}\right| + C$

89. $\displaystyle\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

90. $\displaystyle\int \frac{du}{u^2\sqrt{u^2\pm a^2}} = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u} + C$

91. $\displaystyle\int u^2\sqrt{u^2+a^2}\,du = \frac{u}{8}(2u^2+a^2)\sqrt{u^2+a^2} - \frac{a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

92. $\displaystyle\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

93. $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u^2}\,du = -\frac{\sqrt{u^2+a^2}}{u} + \ln(u+\sqrt{u^2+a^2}) + C$

94. $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u+\sqrt{u^2-a^2}| + C$

95. $\displaystyle\int \frac{u^2}{\sqrt{u^2+a^2}}\,du = \frac{u}{2}\sqrt{u^2+a^2} - \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

96. $\displaystyle\int \frac{u^2}{\sqrt{u^2-a^2}}\,du = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

INTEGRALS CONTAINING $(a^2+u^2)^{3/2}$, $(a^2-u^2)^{3/2}$, $(u^2-a^2)^{3/2}$ ($a > 0$)

97. $\displaystyle\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

98. $\displaystyle\int \frac{du}{(u^2\pm a^2)^{3/2}} = \pm\frac{u}{a^2\sqrt{u^2\pm a^2}} + C$

99. $\displaystyle\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$

100. $\displaystyle\int (u^2+a^2)^{3/2}\,du = \frac{u}{8}(2u^2+5a^2)\sqrt{u^2+a^2} + \frac{3a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

101. $\displaystyle\int (u^2-a^2)^{3/2}\,du = \frac{u}{8}(2u^2-5a^2)\sqrt{u^2-a^2} + \frac{3a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{a+bu}$ OR ITS RECIPROCAL

102. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C$

103. $\displaystyle\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(15b^2u^2-12abu+8a^2)(a+bu)^{3/2}+C$

104. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$

105. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$

106. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu}+C$

107. $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int\frac{u^{n-1}\,du}{\sqrt{a+bu}}$

108. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C & (a>0)\\[2ex] \dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bu}{-a}}+C & (a<0)\end{cases}$

109. $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int\frac{du}{u^{n-1}\sqrt{a+bu}}$

110. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu}+a\int\frac{du}{u\sqrt{a+bu}}$

111. $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u^n} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int\frac{\sqrt{a+bu}\,du}{u^{n-1}}$

POWERS OF u MULTIPLYING OR DIVIDING $\sqrt{2au-u^2}$ OR ITS RECIPROCAL

112. $\displaystyle\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

113. $\displaystyle\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2}+\frac{a^3}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

114. $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u} = \sqrt{2au-u^2}+a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

115. $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u^2} = -\frac{2\sqrt{2au-u^2}}{u}-\sin^{-1}\left(\frac{u-a}{a}\right)+C$

116. $\displaystyle\int \frac{du}{\sqrt{2au-u^2}} = \sin^{-1}\left(\frac{u-a}{a}\right)+C$

117. $\displaystyle\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au}+C$

118. $\displaystyle\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2}+a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

119. $\displaystyle\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2}+\frac{3a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

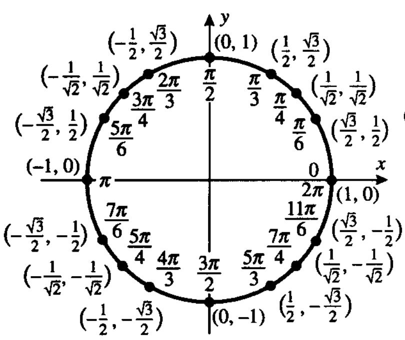
INTEGRALS CONTAINING $(2au-u^2)^{3/2}$

120. $\displaystyle\int \frac{du}{(2au-u^2)^{3/2}} = \frac{u-a}{a^2\sqrt{2au-u^2}}+C$

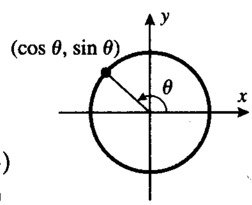
121. $\displaystyle\int \frac{u\,du}{(2au-u^2)^{3/2}} = \frac{u}{a\sqrt{2au-u^2}}+C$

THE WALLIS FORMULA

122. $\displaystyle\int_0^{\pi/2}\sin^n u\,du = \int_0^{\pi/2}\cos^n u\,du = \frac{1\cdot3\cdot5\cdot\cdots\cdot(n-1)}{2\cdot4\cdot6\cdot\cdots\cdot n}\cdot\frac{\pi}{2}\left(\begin{matrix}n\text{ an even}\\ \text{integer and}\\ n\ge 2\end{matrix}\right)$ or $\displaystyle\frac{2\cdot4\cdot6\cdot\cdots\cdot(n-1)}{3\cdot5\cdot7\cdot\cdots\cdot n}\left(\begin{matrix}n\text{ an odd}\\ \text{integer and}\\ n\ge 3\end{matrix}\right)$



TRIGONOMETRY REVIEW



PYTHAGOREAN IDENTITIES

$\sin^2\theta+\cos^2\theta=1 \qquad \tan^2\theta+1=\sec^2\theta \qquad 1+\cot^2\theta=\csc^2\theta$

SIGN IDENTITIES

$\sin(-\theta)=-\sin\theta \qquad \cos(-\theta)=\cos\theta \qquad \tan(-\theta)=-\tan\theta$

$\csc(-\theta)=-\csc\theta \qquad \sec(-\theta)=\sec\theta \qquad \cot(-\theta)=-\cot\theta$

COMPLEMENT IDENTITIES

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta \qquad \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$

$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

SUPPLEMENT IDENTITIES

$\sin(\pi-\theta)=\sin\theta \qquad \cos(\pi-\theta)=-\cos\theta \qquad \tan(\pi-\theta)=-\tan\theta$

$\csc(\pi-\theta)=\csc\theta \qquad \sec(\pi-\theta)=-\sec\theta \qquad \cot(\pi-\theta)=-\cot\theta$

$\sin(\pi+\theta)=-\sin\theta \qquad \cos(\pi+\theta)=-\cos\theta \qquad \tan(\pi+\theta)=\tan\theta$

$\csc(\pi+\theta)=-\csc\theta \qquad \sec(\pi+\theta)=-\sec\theta \qquad \cot(\pi+\theta)=\cot\theta$

ADDITION FORMULAS

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\displaystyle\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\displaystyle\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

DOUBLE-ANGLE FORMULAS

$\sin 2\alpha=2\sin\alpha\cos\alpha \qquad \cos 2\alpha=2\cos^2\alpha-1$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha \qquad \cos 2\alpha=1-2\sin^2\alpha$

HALF-ANGLE FORMULAS

$\displaystyle\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$