Interpolation of Operators

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Preface

Three classical interpolation theorems form the foundation of the modern theory of interpolation of operators. They are the M. Riesz convexity theorem (1926), G. O. Thorin's complex version of Riesz' theorem (1939), and the J. Marcinkiewicz interpolation theorem (1939). The ideas of Thorin and Marcinkiewicz were reworked some twenty years later into an abstract theory of interpolation of operators on Banach spaces and more general topological spaces. Thorin's technique has given rise to what is now known as the complex method of interpolation, and Marcinkiewicz' to the real method. Both have found widespread application, have extensive literatures attached to them, and remain very much alive as subjects of current research.

This is a book about the real method of interpolation. Our goal has been to motivate and develop the entire theory from its classical origins, that is, through the theory of spaces of measurable functions. Although the influence of Riesz, Thorin, and Marcinkiewicz is everywhere evident, the work of G. H. Hardy, J. E. Littlewood, and G. Pólya on rearrangements of functions also plays a seminal role. It is through the Hardy-Littlewood-Pólya relation that spaces of measurable functions and interpolation of operators come together, in a simple blend which has the capacity for great generalization. Interpolation between L^1 and L^∞ is thus the prototype for interpolation between more general pairs of Banach spaces. This theme airs constantly throughout the book.

The theory and applications of interpolation are as diverse as language itself. Our goal is not a dictionary, or an encyclopedia, but instead a brief biography of interpolation, with a beginning and an end, and (like interpolation itself) some substance in between.

The book should be accessible to anyone familiar with the fundamentals of real analysis, measure theory, and functional analysis. The standard advanced undergraduate or beginning graduate courses in these disciplines should suffice. The exposition is essentially self-contained.

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