Review of Calculus I

Douglas Meade, Ronda Sanders, and Xian Wu

Department of Mathematics

Overview

The fundamental operations of calculus are limits, derivatives, and integrals (definite and indefinite). The goal of this week's lab is to review those basic operations using Maple.

Maple Essentials

- The *DerivativeDrill*, *AntiDerivativeDrill*, and *IntBySub* maplets are useful tools to help you practice your differentiation and integration skills. They are also useful to check any derivative or indefinite integral that you compute by hand. These maplets are available from USC on the course website: `http://www.math.sc.edu/calclab/142L-S13/`

- Maple commands introduced in this lab:

Command	Description
`:=x->`	define a function of x. eg: `f:=x->sqrt(x);` `g:=t->2*sin(t);`
`limit`	compute limits or sided limits: `limit(f(x),x=a);` finds the limit, $\lim_{x \to a} f(x)$. `limit(f(x),x=a,right);` finds the one-sided limit from right, $\lim_{x \to a^+} f(x)$. `limit(f(x),x=a,left);` finds the one-sided limit from left, $\lim_{x \to a^-} f(x)$.
`diff`	compute derivatives of an expression: `diff(f(x), x);` finds the derivative of $f(x)$ with respect to x. `diff(f(x), x$n);` finds the nth derivative of $f(x)$.
`int`	compute definite and indefinite integrals: `int(f(x),x);` evaluates the indefinite integral, $\int f(x)\ dx$; `int(f(x),x=a..b);` evaluates the definite integral, $\int_a^b f(x)\ dx$.
`plot`	plot one or more functions on a specified window `plot(f(x), x=a..b);` plots the graph of $y = f(x)$ for $a < x < b$; `plot([f(x),g(x)], x=a..b);` graphs two functions in a single plot
`simplify`	`simplify(f(x));` simplifies $f(x)$

Note that Maple's **int** command does not include any constants of integration. Whenever you evaluate an indefinite integral, do not forget to include a constant of integration (+C) in your answer.

- Basic Functions and Expressions: Maple uses `sqrt(x)` for $\sqrt{x}$, `abs(x)` for $|x|$, `exp(x)` for e^x, and `x^n` for x^n. Other basic functions can be typed in as what they are but you must carefully use ( ) to group together and match up expressions as needed. You must also type in * when multiplication is presented. For example, you need to type in `sin(x)` instead of `sinx` for $\sin x$ and `2*x/(x+y)` instead of `2x/x+y` for $\frac{2x}{x+y}$. Finally, do not forget to type in ; at the end of a line that is to be executed.

- Palettes: The **Expression** palette can be used instead of typing the full Maple commands for many functions and operations. The **Common Symbols** palette contains other symbols, including π (`Pi`) and ∞ (`infinity`).

Related Course Material/Preparation

Review the basic terminology and properties of limits, derivatives, and integrals.

Activities

1. Use the *DerivativeDrill* maplet with polynomial, trigonometric, exponential, and logarithmic functions that utilize the power, product, quotient, and chain rules. Start with only one rule per problem. As you gain confidence, allow up to 2 rules in each problem.

2. Use the *Anti-Derivative Drill* maplet to practice your skills finding the anti-derivative. Add Trigonometric, Exponential, and Logarithmic functions. Don't forget +C!

3. Use the *Integration by Substitution* maplet to practice your skills finding the anti-derivative when you must first rewrite the problem.

   Note: Many of the problems below can be solved in Maple using different interface methods: typing in a command directly, right-clicking over the expression, or using palettes. Try and/or combine different methods when possible.

4. Use Maple to compute the derivative of the following functions:

   (i) $f(x) = x^3 \ln(x)$ $\qquad$ (ii) $g(x) = \dfrac{\arctan(x)}{\sqrt{x^2 + 1}}$ $\qquad$ (iii) $h(x) = \dfrac{1 - \ln(x)}{1 + \ln(x)}$

   Notice that if you use a right-click to obtain the derivative the resulting derivative will be defined as a function. If you use the `diff` command the resulting derivative will be an expression.

5. Use Maple to compute the first and 100th derivatives of $f(x) = x\sin(2x)$.

6. Use Maple to evaluate the following indefinite and definite integrals:

   (i) $\displaystyle\int \frac{x + 1}{(x - 2)^2}\ dx$ $\qquad$ (ii) $\displaystyle\int_{-1}^{3} xe^{-x^2}\ dx$ $\qquad$ (iii) $\displaystyle\int_{0}^{\pi/2} \left|\frac{1}{2} - \cos x\right|\ dx$

7. Use Maple to evaluate the following limits:

   (i) $\displaystyle\lim_{x \to 0} \frac{x^2 \arccos(x)}{(\sin(2x))^2}$ $\qquad$ (ii) $\displaystyle\lim_{n \to \infty} (1 + 3/n)^{2n}$ $\qquad$ (iii) $\displaystyle\lim_{t \to 1^+} \tan\left(\frac{\pi}{2}t\right)$

8. Let

$$f(x) = \ln\left(\frac{(x^2 + 2)^2}{\sqrt{4x^2 + 1}}\right).$$

   In a Maple worksheet:

   (a) Define the function using proper Maple notation.

   (b) Use right-clicking to find the first and second derivatives of $f(x)$. Assign the derivatives to names using `:=` with appropriate label references.

   > **Hint:** *Reasonable names for these quantities would be* `df` *and* `ddf`.

   (c) Find $f(1)$, $f'(1)$, and $f''(1)$. Explain what each of these values tells you about the function $f(x)$ when $x = 1$.

   (d) Plot the graph of $y = f'(x)$ on the interval $-4 < x < 4$ and estimate the intervals on which $y = f(x)$ is increasing.

   (e) Plot the graph of $y = f''(x)$ on the interval $-4 < x < 4$ and estimate the intervals on which $y = f(x)$ is concave up.

   (f) Create a single plot containing the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$ for $-4 < x < 4$. Did you estimate correctly in (d) and (e)?

   (g) Don't forget to give your graph a title and legend and make sure the curves can be identified on a black/white copy. (Review Calc I Lab C if necessary.) You can achieve this using commands or by right-clicking on your graph to see the context menu. Remember if you use right-clicking that your changes will be lost if you re-execute.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the **beginning** of next week's lab.