

Applications of Integration I: Areas Between Curves

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Overview

This lab will help to develop your understanding of the definite integral as area for non-negative functions. In this lab you will explore how to use Maple to visualize a given region, determine points of intersection if necessary, and determine its area. You will discover that for some regions it is best to regard x as a function of y .

Maple Essentials

New Maple commands introduced in this lab include:

Command	Description
<code>fsolve</code>	numerically solves one or more equations for their unknowns <code>fsolve(x^2-4=0);</code> returns the values -2.0, 2.0 *To find only real solutions in an interval, specify the interval. <code>fsolve(x^2-4=0, x=-3..1);</code> returns only -2.0
<code>int(f(x), x=a..b);</code>	evaluates $\int_a^b f(x)dx$

The *Area of a Region by Slicing* maplet is available from the course website:

<http://www.math.sc.edu/calclab/142L-S13/labs> → [Area of a Region by Slicing](#)

This maplet demonstrates finding the area of a region by slicing. The student may generate a random example or modify or make their own problem.

Preparation

In the case where both $f(x)$ and $g(x)$ are positive, it is very easy to see that if we want to find the area between $f(x) \geq g(x)$ (both continuous on $[a, b]$) we would do the following:

$$A = \int_a^b f(x) dx - \int_a^b g(x) dx = \int_a^b (f(x) - g(x)) dx.$$

In general, for a given region with top curve y_T and bottom curve y_B we can draw a typical approximating rectangle with height $(y_T - y_B)$ and width Δx and calculate the area as follows:

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n (y_T - y_B) \Delta x = \int_a^b (y_T - y_B) dx$$

Sometimes regions are best treated by regarding x as a function of y . If a region is bounded by curves with equations $x = f(y)$, $x = g(y)$, $y = c$, and $y = d$, where f and g are continuous and $f(y) \geq g(y)$ for $c \leq y \leq d$ then its area is

$$A = \int_c^d (f(y) - g(y)) dy.$$

In general, for a given region with right boundary x_R and left boundary x_L we can draw a typical approximating rectangle with height Δy and width $(x_R - x_L)$ and calculate the area as follows:

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_R - x_L) \Delta y = \int_c^d (x_R - x_L) dy$$

Related Course Material

Section 6.1 in Stewart. Section 7.1 in CalcLabs.

Activity 1

Find the area of the region enclosed between the curves $y = x^2$ and $y = x + 6$.

- We will first assign our functions as $f(x)$ and $g(x)$.
`> f:=x-> x^2;`
`> g:=x-> x+6;`
- Next, we plot the functions to get an idea of the region we are considering.
`> plot([f(x),g(x)], x=-5..5, y=-10..10, color=[red, blue]);`
- We find the intersection points using `fsolve`.
`> fsolve(f(x)=g(x));`
- We then evaluate the integral to find the area. Notice for this example $g(x)$ is the top curve and $f(x)$ is the bottom curve.
`> Area:=int(g(x)-f(x), x=-2..3);`

Activity 2

Find the area of the region enclosed between $y = -0.128x^3 + 1.728x^2 - 5.376x + 2.864$ and $y = \ln x$.

- We will first assign our functions as $f(x)$ and $g(x)$.
`> f:=x-> -0.128*x^3+1.728*x^2-5.376*x+2.864;`
`> g:=x-> ln(x);`
- Next, we plot the functions to get an idea of the region we are considering. Be careful with the window this time. There are two separate areas enclosed by the functions.
`> plot([f(x),g(x)], x=0..10, y=-10..10, color=[red, blue]);`
- We can see from the graph that there are three points of intersection, but `fsolve` (without a specified interval) will only give one, so we will write a separate command to find and assign each intersection point.
`> a:= fsolve(f(x)=g(x), x=0..2);`
`> b:= fsolve(f(x)=g(x), x=2..6);`
`> c:= fsolve(f(x)=g(x), x=8..10);`
- For our first area, notice that between a and b , $g(x)$ is the top curve and $f(x)$ is the bottom curve.
`> A1:=int(g(x)-f(x), x=a..b);`
- For our second area, notice that between b and c , $f(x)$ is the top curve and $g(x)$ is the bottom curve.
`> A2:=int(f(x)-g(x), x=b..c);`
- We find our total area by adding $A1$ and $A2$.
`> Area:= A1 + A2;`

Activity 3

Find the area of the region enclosed between $x = y^2$ and $x = y + 2$.

- We will first assign our expressions as $f(y)$ and $g(y)$.
`> f:=y-> y^2;`
`> g:=y-> y+2;`
- We can use the `fsolve` command to find the points of intersection.
`> f(y)=g(y);`
- Once you have the points of intersection, you can use the *Area of a Region by Slicing* maplet to get a clear picture of the region.
- We then evaluate the integral to find the area. Notice for this example $g(y)$ is the right boundary and $f(y)$ is the left boundary. Also, we are integrating with respect to y .
`> Area:=int(g(y)-f(y), y=-1..2);`

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely.