Applications of Integration II: Volume By Definite Integral

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Overview

The topic of Volume of Revolution (Stewart §6.2 and §6.3, CalcLabs §7.2) is often difficult for students to grasp. With different methods, formulations, and rotating axes, this topic may appear confusing at first and adequate practice is a must. The objective of this lab is to introduce visual and interactive Maple tools to help with Volume of Revolution problems. Related Maple commands are also introduced.

Maple Essentials

- The *Volumes of Revolution* maplet is available from the course website:

  http://www.math.sc.edu/calclab/142L-S12/labs → <u>Volumes of Revolution</u>.

  This maplet is designed to help you to practice individual steps in computing the volume of revolution of a computer-generated or user-specified region (click on `New Problem` or `Modify or Make Your Own Problem`, respectively). After getting or entering the problem, your first step (`Step 1`) is to choose the appropriate method: `Disks`, `Washers`, or `Cylinders`. You then enter the integral for the volume in `Step 2`. You may click on `Hint` or `Show Template` if you need help to get started. After setting up the correct integral, you can enter your evaluation to check it or let the maplet evaluate the integral by clicking on `Show` in `Step 3`. An important feature of this maplet is its visualization tool. Please click on different boxes in the `Animation` window to try them out. You can rotate the plot with your mouse if you wish.

  This is also a great tool to use when you want to see the plot of the solid or want to check your answer to a problem. It shows not only the answer but also the formulation. However, you need to make sure that your region is entered correctly.

- The *Volumes by Slicing* maplet is available from the course website:

  http://www.math.sc.edu/calclab/142L-S12/labs → <u>Volumes by Slicing</u>.

  This maplet deals with volumes by the method of slicing and has similar features as the *Volumes of Revolution* maplet just introduced. It can handle solids with the most common types of regions as bases but may not handle every problem of this type.

- Important Maple commands introduced in this lab:

Command/Example	Description
`with(Student[Calculus1]):`	loads a special Maple package.

`VolumeOfRevolution(`*top*, *bot*, *x=a..b*, *options*`);`
 Examples:
 `VolumeOfRevolution(x,x^2,x=0..1,output=plot);`
 `VolumeOfRevolution(arcsin(x),0,x=0..1,output=integral);`
 `VolumeOfRevolution(x,x^2,x=0..1,axis=vertical,output=integral);`
 Note: *You need to load the* `Student[Calculus1]` *package before using this command (see command* `with` *above).*
	visualize a solid of revolution of a region or find the definite integral for its volume

The general syntax for *visualizing* a solid of revolution of a region bounded by $y = top$ from above and $y = bot$ from below over the interval $a \leq x \leq b$ is:

> `VolumeOfRevolution(`*top*`, `*bot*`, x=a..b, output=plot);`

Note: *When the bottom edge of the region is the x-axis ($y = 0$), the second argument (bot) can be omitted, but this is not advised.*

To get the *definite integral* for the volume instead of the plot, change the option `output=plot` to `output=integral` in the above command. To evaluate the integral obtained, right-click and choose the option **Evaluate**.

The default axis of revolution is the x-axis ($y = 0$), and the volume is computed by the method of disks/washers. If the axis of revolution is the y-axis ($x = 0$), include `axis=vertical` as an additional argument in the above command. In this case, the volume is calculated by the method of shells. If the axis of revolution is a line other than the x-axis or the y-axis, say, $y = -1$, include `distancefromaxis=-1` as an additional option in the above command.

Activities

1. Use the *Volume of Revolution* maplet to test your formulation of the definite integral for the volume of revolution for each of following solids formed by rotating a region about the x-axis. Evaluate the integral if possible and use the maplet to check your answer. (You may want to first graph the functions to get an idea of what you are rotating.)

   (a) $y = x$, $y = 2 - x^2$, $x = 0$, $x = 1$. The method of disks/washers should be used. Why?

   (b) $y = x^2$, $y = 2 - x$, $y = 0$. The method of shells should be used. Why?

   (c) $y = x^2$, $y = x^3$. Try both methods.

   (d) $y = \sqrt{x}$, $y = 2 - x$, $y = 0$. Which method should be used?

   (e) $y = \sqrt{x}$, $y = 2 - x$, $x = 0$. Which method should be used?

   General Steps:

   i. Sketch the region and find intersections if needed (use Maple if you have to). Identify the rotating axis.

   ii. Decide which method to use and then choose the variable of integration accordingly. **The rule is to select the same variable of integration as the rotating axis with the method of disks/washers but the different variable with the method of shells.**

   iii. Start the maplet and click on `Modify or Make Your Own Problem` to enter the problem. **Note:** *You need to convert equations of curves to express x as a function of y if the variable of integration is y.*

   iv. Select the method to use (in `Step 1`) and enter your integral to check (in `Step 2`). If you need the general formula, click on `Show Template`.

   v. Evaluate the integral by hand and use the Maplet to check your answer. You may also want to try the animation tool.

2. Repeat Activity 1 for each of the following solids formed by rotating a region about the y-axis.

   (a) $y = \sin(x)$, $y = \cos(x)$, $x = 0$, $x = \pi/4$. The method of shells should be used. Why?

   (b) $y = 1/x$, $x = 2$, $y = 2$. Try both methods. Which one is better?

   (c) $y = x^2$, $y = x^3$. Try both methods and compare them to (c) of Activity 1.

3. Repeat at least one example from each of Activity 1 and Activity 2 using the `VolumeOfRevolution` command in a Maple worksheet. Remember to execute the command: `with(Student[Calculus1]):` to load the package first.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely.