

# Applications of Integration III: Area of a Surface of Revolution

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## Overview

A surface of revolution is formed when a curve is rotated about a line. Such a surface is the lateral boundary of a solid of revolution of the type discussed in last week's lab on Volume by Definite Integral. The objective of this lab is to introduce visual and interactive Maple tools to help with Area of a Surface of Revolution problems. Related Maple commands are also introduced.

## Maple Essentials

- Important Maple commands introduced in this lab:

Command/Example	Description
<code>with(Student[Calculus1]):</code>	loads a special Maple package.
<code>SurfaceOfRevolution(f(x), x=a..b, options);</code> <b>Examples:</b> <code>SurfaceOfRevolution(x^2, x=0..1, output=plot);</code> <code>SurfaceOfRevolution(arcsin(x), x=0..1, output=integral);</code> <code>SurfaceOfRevolution(x^2, x=0..1, axis=vertical, output=integral);</code> <b>Note:</b> You need to load the <code>Student[Calculus1]</code> package before using this command (see command with above).	visualize a surface of revolution of a region or find the definite integral for its surface area

The general syntax for *visualizing* a surface of revolution generated by rotating the expression  $f(x)$  from  $a$  to  $b$  around the horizontal axis:

```
> SurfaceOfRevolution(f(x), x=a..b, output=plot);
```

To get the *definite integral* for the surface area instead of the plot, change the option `output=plot` to `output=integral` in the above command. To evaluate the integral obtained, right-click and choose the option **Evaluate**.

The default axis of revolution is the  $x$ -axis ( $y = 0$ ). If the axis of revolution is the  $y$ -axis ( $x = 0$ ), include `axis=vertical` as an additional argument in the above command.

## Activities

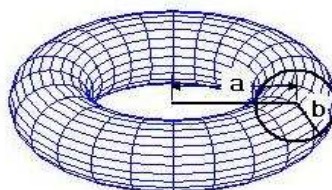
1. Use the `SurfaceOfRevolution` command to find the area of the curve obtained by rotating the curve  $y = x^3$ ,  $0 \leq x \leq 2$  about the  $x$ -axis. (Remember you can use the option `output=plot` to first visualize the surface.)
2. Repeat Activity 1 rotating about the  $y$ -axis.

## Assignment

This week's assignment is to complete the project **The Skimpy Donut** on page 216 of CalcLabs with Maple. **This project is 20% of your Maple grade.** The next part of this lab will help you get started.

### Starting The Skimpy Donut Project

Consider the following donut. Could we create such a solid by rotating a region about the  $x$ -axis or the  $y$ -axis?



Notice that the donut has the shape of a torus which is formed by revolving the circle with radius  $b$  and center  $(a, 0)$  around the  $y$ -axis. Recall from Algebra that the equation of such a circle would be

$$(x - a)^2 + y^2 = b^2.$$

The GETFAT Donut company currently makes donuts with  $a = 5\text{cm}$  and  $b = 3\text{cm}$ . We will determine the resulting volume of such a donut. Also, the company coats its donuts with a thin layer of chocolate icing, so we will compute the surface area of such a donut to determine the amount of chocolate necessary.

- The GETFAT Donut company's current donut would result by revolving a circle with equation  $(x - 5)^2 + y^2 = 9$  about the  $y$ -axis.
- **Step 1: Determining the Volume** Since we are rotating about the  $y$ -axis, we can determine the volume using the method of washers and integrating with respect to  $y$ . To use this method, we will need to solve the equation of the circle for  $x$  and determine the right boundary ( $x_R$ ) and the left boundary ( $x_L$ ) of the circle. We can then use the `VolumeOfRevolution` command to visualize the solid and determine the volume.
 

```
> VolumeOfRevolution( x_R, x_L, y=-3..3, output=plot, scaling=constrained );
> VolumeOfRevolution( x_R, x_L, y=-3..3, output=integral );
```

**Note:** The other option for determining the volume is to use the method of shells and integrate with respect to  $x$ . Maple encounters an error when using the `VolumeOfRevolution` command for this method. You can determine the integral by hand or use the `Volumes Of Revolution` maplet if you want to use this method.
- **Step 2: Determining the Surface Area** To determine the surface area, we will need to solve the equation of the circle for  $y$  and determine the top curve ( $y_T$ ) and the bottom curve ( $y_B$ ) of the circle. We can then use the `SurfaceOfRevolution` command (twice) to visualize and determine the total surface area of the donut.
 

```
> SurfaceOfRevolution(y_T, x=2..8, axis=vertical, output=plot, scaling=constrained);
> SurfaceOfRevolution(y_B, x=2..8, axis=vertical, output=plot, scaling=constrained);
> SurfaceOfRevolution( y_T, x=2..8, axis=vertical, output=integral )
    + SurfaceOfRevolution( y_B, x=2..8, axis=vertical, output=integral );
```

For the project, you will compute a volume of revolution, a surface area of revolution, and solve a max/min problem. Please see the next page for helpful hints.

### *Project Preparation*

Read the *Project Report Guidelines* before beginning your project. Remember, you are to turn in a neat and complete project report. Any figures should have a title and be properly referenced in the report. A complete project report should include all necessary equations and information. You should hand in a Word document, not just a Maple worksheet. Remember that your project report should have only one author. Do not copy another student's work.

### *Project Helpful Hints*

1. For the project, you will compute a volume of revolution and surface of revolution for the generalized donut. These computations are similar to the ones at the end of the lab, but you will need to use the command  
`> value( Insert label ) assuming  $b > 0, a > b$ ;`  
to find the value of your integrals.
2. You should include the integrals for Volume and Surface Area within your report.
3. Once you have formulas for volume and surface area (also to be included in your report), you will determine the minimum surface area with a fixed volume of  $90\pi^2$ . You can do this by hand or review the Fall 2010 Calculus I lab Optimization

<http://www.math.sc.edu/calclab/141L-F10/labs/Optimization.pdf>

for help solving using Maple. It will be very important for you to consider the allowable range for  $a$  and  $b$ .

4. In your report, remember to include the relative change (or percent change) in the surface area as this represents potential savings or extra costs in the amount of chocolate.