Integration Methods II: Partial Fractions and Trig Substitutions

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Overview
The objective of this lab is to use Maple to practice integration skills using partial fraction decomposition and trigonometric substitutions.

Maple Essentials

1. Important Maple commands introduced in this lab:

<table>
<thead>
<tr>
<th>Command/Example</th>
<th>Description</th>
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<tbody>
<tr>
<td>convert(f(x), parfrac, x);</td>
<td>Converts a rational function ( f(x) ) into its partial fraction form.</td>
</tr>
<tr>
<td>Example: convert(x/((x^2+1)*(x-2)^3)),parfrac,x);</td>
<td></td>
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<tr>
<td>completesquare(f(x),x);</td>
<td>Completes the square to rewrite a quadratic expression.</td>
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<tr>
<td>Example: completesquare(x/(x^2+2*x+2),x);</td>
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<tr>
<td>Note: You need to load the student package before using this command.</td>
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Note: You may omit the last \( x \) in the above commands if the independent variable is obvious.

2. The Trig Substitution maplet is available on the course website:
http://www.math.sc.edu/calclab/142L-F13/labs → Trig Substitution
This maplet is designed to help you practice the steps involved in integration using trigonometric substitutions. Click on New Integral to have the maplet generate a problem for you to practice. Next, identify the type of trigonometric substitution and follow the steps to solve the problem. (You will need a pencil and paper to do all the steps yourself.) You can click Hint or Show if you need help or want to skip a step.

3. The Partial Fractions: General Decomposition maplet is available on the course website:
http://www.math.sc.edu/calclab/142L-F13/labs → Partial Fractions: General Decomposition
This maplet is designed to help you practice partial fraction decomposition. Click on New Function to have the maplet generate a problem for you to practice. Then add terms one at a time until you have the general form for the decomposition. Once you are confident, click on the Include Completing the Square box to make the decomposition more difficult.

4. The Partial Fractions: Evaluating the Integral maplet is available on the course website:
http://www.math.sc.edu/calclab/142L-F13/labs → Partial Fractions: Evaluating the Integral
This maplet is designed to help you practice evaluating the integral after using partial fraction decomposition. Click on Modify or Make Your Own Problem to enter an integral. Click on Show Partial Fraction Expansion. Then evaluate the expanded integral and check your work.
Related course material

Stewart text: §7.3 and §7.4. Calculus text: §6.5.

Activities

1. Launch the Partial Fractions: General Decomposition maplet from the course web page and do a few practice problems.

2. Follow the steps below to integrate the following rational functions:

   (a) \( f_1(x) = \frac{1}{(x+a)(x+b)} \). (Page 482, ex. 14)

   (b) \( f_2(x) = \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} \). (Page 482, ex. 15)

   (c) \( f_3(x) = \frac{x^3 + 4}{x^2 + 4} \). (Page 482, ex. 21)

   (d) \( f_4(x) = \frac{x^2 - 2x - 1}{(x - 1)^2(x^2 + 1)} \). (Page 482, ex. 28)

   (e) \( f_5(x) = \frac{x + 4}{x^2 + 2x + 5} \). (Page 482, ex. 29)

   (f) \( f_6(x) = \frac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2} \). (Page 482, ex. 38)

Steps:

   The following steps would be used to integrate \( f_1(x) \).

   (a) Load the maple student package.
      \[
      > \text{with(student)};
      \]

   (b) Assign the function using the arrow notation.
      \[
      > f1 := x \rightarrow 1/((x+a)*(x+b));
      \]

   (c) Write down the general form for the partial fraction decomposition of the function by hand. Then use Maple to complete the expansion and compare it to your general form.
      \[
      > \text{convert(f1(x), parfrac, x)};
      \]

   (d) Use a pencil and paper to integrate the function. Then use Maple to complete the integration and check your answer.
      \[
      > \text{int(f1(x),x)};
      \]

   (e) You can also use the Partial Fractions: Evaluating the Integral maplet to check your integration.

3. Launch the Trig Substitution maplet from the course web page and do a few practice problems. Remember, you will need a pencil and paper to complete all the steps by yourself.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the beginning of next week’s lab.