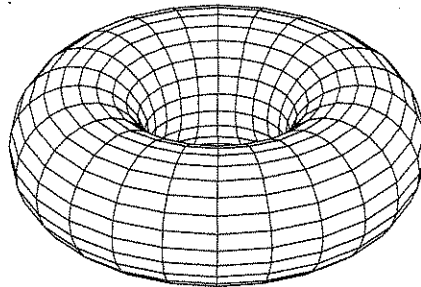


12.18 The Skimpy Donut

GOAL: In this project, you will compute a volume of revolution, a surface area of revolution, and solve a max/min problem.

ASSIGNMENT: The GETFAT Donut company makes donuts with a thin layer of chocolate icing. The company decides to cut costs by minimizing the amount of chocolate icing used on each donut without shrinking the volume of the donut.



The donut has the shape of a torus which is formed by revolving the circle $(x - a)^2 + y^2 = b^2$ around the y -axis. (Here b is the radius of the circle and a is the distance from the center of the hole to the center of the circle.) At present the company makes donuts with $a = 5$ cm and $b = 3$ cm. The problem, then, is to determine the dimensions of the donut with the same volume that minimize the surface area. Follow these steps:

1. Compute the volume V of the donut as a function of a and b using the technique of either washers or cylindrical shells. As a check, verify the volume of the donut with $a = 5$ and $b = 3$ is $90\pi^2 \approx 888$ cm³.

HINT: First execute `assume(b>0, a>b)`;

2. Compute the surface area of the donut as a function of a and b either by solving the equation of the circle for x or y or by parametrizing the circle as $x = a + b \cos t$, $y = b \sin t$. As a check, verify the surface area of the donut with $a = 5$ and $b = 3$ is $60\pi^2 \approx 592$ cm².

3. With the volume fixed at $90\pi^2$, find the dimensions a and b of the donut which minimize the surface area of the donut.

NOTE: For this problem, you will have to determine the range of allowable a and b .

4. Is there a maximum surface area for the given volume? (So the company could advertise extra chocolate.)

5. Write a report to the CEO summarizing your recommendations (including the percent savings or percent extra cost). Anything you say in this report must be documented in an appendix of Maple computations for the engineers.