Related Rates

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Overview

Related Rates problems involve finding the rate at which a quantity changes by relating that quantity to other quantities whose rates of change are known. Related rates problems are one of the principle applications of the Chain Rule for differentiation. In this lab, you will learn to use Maple to assist in solving related rates problems.

Maple Essentials

- The *RelatedRates* maplet is available from the course website:

  `http://www.math.sc.edu/calclab/141L-S10/labs/` → RelatedRates

- The new Maple commands introduced in this lab are:

Command	Description	Example
`solve`	solves one or more equations for their unknowns	`solve(2x+5y-7=0, y);`
`diff`	differentiate	`diff(f(x), x);`
`subs`	substitute the given values into an expression	`subs({r=2, h=4},Pi*r^2*h);`

Preparation

Review §3.9 Related Rates in Stewart and §5.1 in *Calclabs with Maple.*

General Steps for Solving Related Rates Problems

The following strategy for solving related rates problems is outlined on Page 243 of Stewart.

1. Read the problem carefully.
   (What rate of change is the problem asking you to identify? What quantities are known?)
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution.
6. Use the Chain Rule to differentiate both sides of the equation with respect to t.
7. Substitute the given information into the resulting equation and solve for the unknown rate.

Activities

1. The *RelatedRates* maplet gives a step-by-step method for solving a related rates problem in which two variables are related by a given equation. Launch the maplet and click **New Problem**.
   (a) For **Step 1**, differentiate the equation with respect to t. Remember that both given variables are time-dependent so you will have to use the Chain Rule. You will need to include `dx/dt` and `dy/dt` in your derivative. Click **check** to have the maplet check your work before continuing to the next step.
   (b) For **Step 2**, substitute the given values for x, y, and the known rate of change. Click **check**.
   (c) For **Step 3**, solve for the unknown rate of change. Click **check**.
2. Solve each of the related rates problems on the back of this page using Maple when necessary.

Example Problem

We will solve Example 1 on page 241 of Stewart together using Maple:
Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?
Steps:

1. You should start every new related rates problem in Maple with a **restart** command. This will prevent confusion and miscalculation if the same variables are used in different problems.
   `> restart;`
2. We are given the rate of change of the volume and asked for the rate of change of the radius, so we need a formula that relates volume and radius. This is easy enough since the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. This equation is entered into Maple using $V(t)$ and $r(t)$ to signify that both quantities are time-dependent.
   `> Veq:=V(t)=4/3*Pi*r(t)^3;`
3. This equation is then differentiated with respect to time.
   `> dVeq:=diff(Veq, t);`
4. For ease of reference in the future, we will assign names to $\frac{dV}{dt}$ and $\frac{dr}{dt}$.
   `> dV:=diff(V(t),t);`
   `> dr:=diff(r(t),t);`
5. Now, we solve for the unknown quantity, $\frac{dr}{dt}$, and substitute in for the known values.
   `> dr:=solve(dVeq, dr);`
   `> subs({r(t)=25, dV=100}, dr);`

Additional Problems

1. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is $16cm^2$? (Ex. 3 page 245 of Stewart)
2. A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing? (Ex. 5 page 245 of Stewart)
3. Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars changing two hours later? (Ex. 15 page 245 of Stewart)
4. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has a height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank. (Ex. 23 on page 246 of Stewart)

Assignment

Exercises 12, 14, and 18 on page 245 of Stewart.