Project: Constructing a Box at Minimal Cost

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Preparation

Be sure to read the *Project Report Guidelines* before beginning your project. Remember, you are to turn in a neat and complete project report. Any figures should have a title and be properly referenced in the report. Do not turn in a Maple worksheet. A complete project report should include all necessary equations and information. Remember that your project report should have only one author. Do not copy another student's work.

The Problem

You have been hired to build a rectangular box with a square base. The volume of this box is to be 1000in^3. You want to make this box at minimal cost. Each of the six faces costs a cents per square inch and gluing each of the 12 edges costs b cents per (linear) inch of length.

1. Find an equation that represents the total cost C as a function of x, the length of one side of the base.
2. Suppose $a = b = 1$. Find the dimensions of the box of minimal cost. How much would this box cost to construct? Use the graph of $C(x)$ to show that you have a minimum.
3. Repeat Step 2 with a and b being the last two distinct nonzero digits of your student ID number.
4. What do you notice about your results? Prove your conclusion using the original formula for $C(x)$ from Step 1. That is, redo Step 2 with a and b being variables. How can you prove that you have a minimum? Find a formula for the minimal cost of the box in terms of a and b.
5. Suppose you decide that you do not need to spend as much on the bottom of the box since no one will care what the bottom looks like. The cheapest material you can find is 1 cent per square inch, so you will use this for the bottom. The four sides and top cost a cents per square inch and gluing the 12 edges costs b cents per (linear) inch of length. Use the graph of $C(x)$ to determine the minimal cost if $a > b$ and a and b are the last two distinct digits of your student ID number (such that both a and b are greater than 1).
   Note: *You will want to use fsolve this time so you can specify an interval.*
6. Repeat Step 5, changing the values of a and b. Is the shape of the optimal box (in Step 5) independent of the values of a and b?

Acknowledgment

This project is based on a project from Edwards & Penny, *Calculus with Analytic Geometry*, 5^{th} Ed.