

# Functions

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## Overview

In this lab, we will use Maple to help us calculate and simplify combinations of functions. The **Domain and Range Finder** will also be introduced to help us practice our skills identifying the domain and range of given functions.

## Maple Essentials

- The *Domain and Range Finder* maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-S10/labs/> → Domain and Range Finder

- New Maple commands introduced in this lab include:

Command	Description
<code>:=</code>	assign a name to a quantity
<code>:=x-&gt;</code>	defines a function with independent variable $x$ <code>f:=x-&gt;a*x+b;</code> assigns $f$ to be the function $f(x) = ax + b$
<code>simplify</code>	applies simplification rules to an expression <code>simplify(f(x));</code> simplifies $f(x)$

## Preparation

Sections 1.1-1.3 in Stewart and Section 2.1 in *CalcLabs with Maple*

## Activities

- Use the **Domain and Range Finder** maplet to practice your skills identifying the domain and range of a given function.
  - From the Calculus I Lab Assignments page under Lab B, click on Domain and Range Finder. You will be prompted for a username and password as these maplets are protected. You should use your Blackboard username and password.
  - Click on the **New Function** button. Enter the domain and range for the given function, then click the **Check Answer** button. Remember that you will use **infinity** for  $\infty$  and you must specify whether an endpoint is included by choosing parentheses or square brackets.

**Note:** If you are having trouble, you can click **Plot It** to view the graph.

- In each of the problems on the next page, you will use the assignment operator (`:=`) together with the arrow notation (`->`) to define each function. Once you have done this, the problems are straightforward. We will work through one example together.

**Note:** Remember that you can use the *Expression and Common Symbols* palettes to avoid typing so much. You may also find the labels useful.

**Example Problems**

For the following problems, let  $f(x) = \frac{1}{x^2 - 4}$  and  $g(x) = \frac{1}{x - 2}$ .

- Find and simplify formulas for  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(fg)(x)$ , and  $(f/g)(x)$ .
- Evaluate the difference quotient  $\frac{f(x+h)-f(x)}{h}$ ,  $h \neq 0$ . Simplify your answer.
- Find and simplify  $(f \circ g)(x)$  and  $(g \circ f)(x)$ .
- Express  $f$  as a composition of two functions:  $f(x) = (f1 \circ f2)(x)$ .

We begin by defining the functions using proper Maple notation.

```
> f:=x-> 1/(x^2-4);
```

```
> g:=x-> 1/(x-2);
```

We can then evaluate each of the operations on the functions.

```
> f(x)+g(x);
```

 To simplify:

```
> f(x)-g(x);
```

 Right-click over the expression.

```
> f(x)*g(x);
```

 Choose **Simplify** from the context menu.

```
> f(x)/g(x);
```

 What is the domain for each of these?

For the difference quotient, remember that you will need parentheses around the numerator.

```
> (f(x+h)-f(x))/h;
```

Evaluate the compositions as follows. Remember to simplify.

```
> f(g(x));
```

```
> g(f(x));
```

**Note:** You can use the notation  $(f@g)(x)$  for composition if you wish.

For the decomposition of  $f(x)$ , you will have to do some work on your own:

```
> f1:=x-> ??;
```

```
> f2:=x-> ??;
```

```
> f1(f2(x));
```

 Did you get  $f(x)$ ?
**Additional Practice**

- Find formulas for  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(fg)(x)$ , and  $(f/g)(x)$  and state their domains.
  - $f(x) = x^3 + 2x^2$ ,  $g(x) = 3x^2 - 1$  (Ex. 29 on Page 44.)
  - $f(x) = 1 + \frac{x}{x+1}$ ,  $g(x) = 2 - \frac{1}{x}$
- Let  $f(x) = 3x^2 - x + 2$ . Find and simplify each of the following. (Ex. 21 on Page 22)
  - $f(2)$
  - $f(a + h)$
  - $f(2a)$
  - $2f(a)$
  - $f(a^2)$
  - $[f(a)]^2$
- Evaluate the difference quotient  $\frac{f(x+h)-f(x)}{h}$ ,  $h \neq 0$ . Simplify your answer.
  - $f(x) = 4 + 3x - x^2$  (Ex. 23 on Page 22)
  - $f(x) = \frac{1}{(x+1)^2}$
- Find  $(f \circ g)(x)$ ,  $(g \circ f)(x)$ ,  $(f \circ f)(x)$ , and  $(g \circ g)(x)$  and their domains.
  - $f(x) = x + \frac{1}{x}$ ,  $g(x) = \frac{x+1}{x+2}$  (Ex. 35 on Page 44)
- Express  $F$  as a composition of two functions:  $F(x) = (f \circ g)(x)$ . (Ex. 41-43 on Page 44)
  - $F(x) = (x^2 + 1)^{10}$
  - $F(x) = \sin(\sqrt{x})$
  - $F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

**Assignment**

With the help of Maple, work out exercises 24 and 26 on page 22 (of Stewart) and exercises 30, 36, and 38 on Page 44 (of Stewart). Write your answers on a sheet of paper to be turned in with your Maple worksheet. Make sure you answer each question completely.