Differentiation Drill

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Overview

This lab uses Maple to help you practice your differentiation skills. Four different user interfaces are introduced.

- The *Derivatives* tutor takes a user-specified function, finds its derivative and plots the function and its derivative on a specified domain. You can also choose to plot the second derivative if you wish.

- The *Graph_df* maplet allows the user to use the graph of the derivative to make conclusions about the properties of the function.

- The *Differentiation Methods* tutor shows the step-by-step application of the differentiation rules involved in finding the derivative of a user-specified function.

- The *DerivativeDrill* maplet allows the user to select types of functions and rules and then generates random problems of this type; the user has to enter the correct derivative. This maplet also provides practice entering mathematical expressions in Maple syntax.

Used properly, these are excellent tools for helping students hone their understanding of derivatives and their differentiation skills.

Maple Essentials

- The new Maple commands introduced in this lab are:

Command	Description
`ln(x)`	(natural) logarithm function, $\ln x$

- The *Derivatives* tutor is started from the Maple user interface under the Tools menu:

Tools → Tutors → Calculus - Single Variable → Derivatives ...

*Be sure to enter the function correctly and specify a reasonable domain by entering appropriate values in the boxes labeled $a =$ and $b =$. The **Display** button displays the formula for the first derivative and plots the function and its derivative on the specified domain. Look at the plot to see how the derivative can be used to determine where the function is increasing and decreasing. Check the box under the second derivative to include the second derivative on the plot. Notice how you can use the second derivative to determine the function's concavity. This is a great tool to use when you need to check your answer to a problem in the text.*

- The *Graph_df* maplet is available from the course website:

`http://www.math.sc.edu/calclab/141L-S10/labs/` → Graph_df

This maplet generates random derivative graph and asks the student to make conclusions about the properties of the function $f(x)$.

- The *Differentiation Methods* tutor is started from the Maple user interface under the Tools menu:

Tools → Tutors → Calculus - Single Variable → Differentiation Methods ...

*This tutor is designed to help you practice the individual steps involved in finding the derivative of a given function. Do not be afraid to use the **Hint** and **Apply Hint** — particularly when the hint suggests using the* Rewrite *rule. Also, you might find it useful to mark the* Constant *and* Constant Multiple *rules as being understood (see the **Understood Rules** menu).*

- The *DerivativeDrill* maplet is available from the course website:

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  `http://www.math.sc.edu/calclab/141L-S10/labs/` $\rightarrow$ <u>DerivativeDrill</u>

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  This maplet generates random differentiation problems using the types of functions and differentiation rules that you select and checks if you correctly find the derivative. Start slowly and build to more complicated expressions as you gain confidence in your differentiation skills.

Preparation

§3.1, §3.2, §3.3, §3.4, and §3.6 of Stewart. Prior to the beginning of this lab, be sure you know the basic differentiation rules.

Activities

Here are a few extended questions to get you started.

1. For each of the functions listed below, find the derivative. Plot the function and derivative(s) on an appropriate window. (See the **Directions** below.) Notice the differences between the original graph and its derivative(s). Examine the plot to see how the derivatives can be used to determine where $f(x)$ is increasing, decreasing, concave up, and concave down.

   (a) $f(x) = x^2 + 4x - 6$
   (b) $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 4$
   (c) $y = \ln(x^2)$
   (d) $y = x \ln x$

   Directions:

   - Launch the *Derivatives* tutor.
   - Enter your function, say $x^2 + 4x - 6$, as $f(x)$ and specify an interval for x, say $a = -5$ and $b = 5$. **Note:** *For the functions involving* $\ln(x)$, *you need to choose a window based on the natural domain of the function.*
   - The tutor shows the original graph in red and the first derivative in blue. Could you tell which graph was the derivative if the curves were not distinguished by color?
   - If you click **Close**, Maple will return your graph to the worksheet.

2. Now use the *Graph_df* maplet to practice using the graph of the derivative $f'(x)$ to make conclusions about the properties of the function $f(x)$.

3. List the rules (in order of application) involved in finding the derivatives of each of the following. (Use the *Differentiation Methods* tutor.)

   (a) $y = x \sin x$
   (b) $y = \ln(x^3 + 3x^2 + 7)$
   (c) $y = 3x^4 + x \cos^2 x$
   (d) $y = \frac{x^2 \sin x}{\cos x}$
   (e) $y = \sin(\cos(x^3))$

4. Use the *DerivativeDrill* maplet with polynomial, trigonometric, exponential, and logarithmic functions that utilize the power, product, quotient, and chain rules. Start with only one rule per problem. As you gain confidence, allow up to 2 rules in each problem.

Assignment

Exercises 40, 42, and 43 on page 181 of Stewart.