

Implicit Differentiation

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Overview

This lab provides experience working with functions defined implicitly.

Maple Essentials

- The new Maple commands introduced in this lab are:

Command	Description	Example
<code>display</code>	combine one or more plots in a single plot; part of the <code>plots</code> package	<code>display([P1,P2], title="My Graph");</code>
<code>implicitdiff</code>	compute derivatives for implicitly-defined functions	Finding $\frac{dy}{dx}$: <code>implicitdiff(eq, y, x);</code> Finding $\frac{d^n y}{dx^n}$: <code>implicitdiff(eq, y, x\$n);</code>
<code>implicitplot</code>	create graph of function defined implicitly; part of the <code>plots</code> package	<code>implicitplot(eq, x=a..b, y=c..d);</code>
<code>pointplot</code>	plots a single point; part of the <code>plots</code> package	<code>pointplot([a,b], symbolsize=15);</code>
<code>fsolve</code>	compute a solution of equations numerically	<code>fsolve({eq1,eq2}, {x,y});</code>
<code>with</code>	loads the contents of a Maple package	<code>with(plots):</code>
<code>eval</code>	evaluates a given expression at a given point	<code>eval(eq, {x=a, y=b});</code>

- The *ImplicitDifferentiation* maplet is available from the course website:

<http://www.math.sc.edu/calclab/141L-F11/labs/> → ImplicitDifferentiation

Preparation

Review §3.5 Implicit Differentiation in Stewart and §4.4 in *Calclabs with Maple*.

Activities

- The curve with equation $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ is called a lemniscate.
 - Find the equation of the tangent line to this curve at the point (3, 1). Then graph the curve, the point, and the tangent line together on one plot with a viewing window of $[-5, 5] \times [-4, 4]$. (Ex. 29 on page 213)
 - Find all points on the lemniscate where the tangent line is horizontal or vertical. (Ex. 39 on page 214)
- Find $\frac{d^2 y}{dx^2}$ and $\frac{d^3 y}{dx^3}$ if y is defined implicitly by $y + \sin y = x$.

Example Problem

We will solve Example 2 on page 209 of Stewart together using Maple:

- Use implicit differentiation to find $\frac{dy}{dx}$ for the Folium of Descartes $x^3 + y^3 = 6xy$.
- Find an equation of the tangent line to the Folium of Descartes at the point $(3, 3)$.
(Then graph the curve, the point, and the tangent line with a viewing window of $[-5, 5] \times [-5, 5]$ as shown in Figure 4 on page 210.)
- At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?
(At what points is the tangent line vertical?)

Steps:

1. First, load the Maple `plots` package. Without the contents of this package, much of what we do today will not work.
`> with(plots):`
2. Assign our equation using `:=`.
`> eq:= x^3 + y^3 = 6*x*y;`
3. Find (and assign) the derivative using implicit differentiation. Since we want $\frac{dy}{dx}$, we input `y` and then `x`.
`> dydx:= implicitdiff(eq, y, x);`
4. Find (and assign) the slope of the tangent line at the point $(3, 3)$.
`> m:= eval(dydx, {x=3, y=3});`
5. Find (and assign) the equation of the tangent line. Remember: $y = m(x - x_1) + y_1$.
`> L:= m*(x - 3) + 3;`
6. Next, write (and assign) commands to plot the curve, the point, and the tangent line. Write the commands separately using `:` so Maple does not display the output yet. (In the first plot command, the option `numpoints=10000` will insure a smooth curve.)
`> P1:= implicitplot(eq, x=-5..5, y=-5..5, numpoints=10000):`
`> P2:= pointplot([3,3], color=green, symbolsize=15):`
`> P3:= plot(L, x=-5..5, y=-5..5, color=blue, linestyle=dash):`
7. Use the `display` command to display the curve, point, and tangent line on a single plot.
`> display([P1, P2, P3], title='Figure 1');`
8. From the graph, we can see that the tangent line would be horizontal at a point located approximately at $(2.5, 3.1)$. To find the point exactly, we need to find a point on the curve where $\frac{dy}{dx} = 0$. We can find this point using `fsolve`.
`> fsolve({eq, dydx=0}, {x=2..3, y=3..4});`
9. From the graph, we can see that the tangent line would be vertical at a point located approximately at $(3.1, 2.5)$. To find the point exactly, we need to find a point on the curve where $\frac{dy}{dx}$ is undefined. That is, a point where the denominator of $\frac{dy}{dx}$ is 0. We can find this point using `fsolve`.
`> fsolve({eq, denom(dydx)=0}, {x=3..4, y=2..3});`

Additional Notes

The ImplicitDifferentiation maplet provides additional practice finding the slope of a curve at a point.

Assignment

With the help of Maple, work out the problems assigned by your lab instructor. Clearly identify your answers on your Maple worksheet. Make sure you answer each question completely. Your assignment is due at the **beginning** of next week's lab.