Differentiation and Tangent Lines

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Overview

This week's lab will provide practice using the limit definition of the derivative and using the derivative to find the equation of the tangent line to a curve.

Maple Essentials

- The *TangentLine* maplet is available from the course website:

  http://www.math.sc.edu/calclab/141L-F10/labs/ → TangentLine

- The Maple commands involved with finding and plotting the tangent line to the graph of a (differentiable) function are:

Command	Description
:=	assignment
:= x ->	function definition
plot	plot one or more expressions

Preparation

Review §2.7 in Stewart and §4.1 and §4.2 in *Calclabs with Maple*.

Limit Definition of the Derivative

Recall the limit definition of the derivative on page 146 of Stewart:
The **derivative of a function f at a number a**, denoted $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Activity

Use Maple and the limit definition of the derivative above to find $f'(a)$ for each of the functions below. (page 151 in Stewart)

a. $f(x) = 3 - 2x + 4x^2$ b. $f(x) = x^4 - 5x$ c. $f(x) = \frac{2x+1}{x+3}$ d. $f(x) = \frac{1}{\sqrt{x+2}}$

Example

It will be easier to first evaluate the difference quotient and then evaluate the limit. Remember that you have to use ctrl-L to insert a label. Say $f(x) = x^2 - 8x + 9$. The steps below are used to find $f'(a)$.

```
> f :=x-> x^2-8*x+9;
> (f(a+h)-f(a))/h;
> limit( (Insert label), h=0);
```

Finding the Equation of the Tangent Line

Page 147 of Stewart states:

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

Recall the point-slope form of the equation of the line:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line and m is the slope of the line. Next, solve the equation for y and we get:

$$y = m(x - x_1) + y_1.$$

Now, we use the substitution $x_1 = a$ and $y_1 = f(a)$ and this becomes:

$$y = m(x - a) + f(a).$$

Finally, we know that the derivative evaluated at $x = a$ is the same as the slope of the tangent line at $x = a$. Thus we get the following formula for the equation of the tangent line at $x = a$:

$$y = f'(a)(x - a) + f(a).$$

Activities

1. The *TangentLine* maplet gives a step by step method for finding the equation of the tangent line to $f(x)$ at the point $x = a$. Launch the maplet and click **New Function**. Follow the prompts to find $f(a)$, $f'(x)$, and $f'(a)$. Then enter the equation of the tangent line as follows:

$$y = f'(a)(x - a) + f(a).$$

   The maplet will check each of your answers and let you know whether you are correct.

2. Use Maple to find the equation of the tangent line to the graph of $f(x)$ at the point $(a, f(a))$ for each of the following functions. Then graph the function and the tangent line on the same axes.
   a. $f(x) = \sqrt{x}$, $a = \frac{1}{4}$ b. $f(x) = \frac{5}{x} + 1$, $a = -2$ c. $f(x) = x^2$, $a = 1$

   d. $f(x) = 2^x$, $a = 3$ e. $f(x) = \cos(x)$, $a = \frac{\pi}{6}$ f. $f(x) = \frac{1}{5x-3}$, $a = 1$

Example Problem

We will solve the following problem together:

Find an equation for the line that is tangent to the graph of the differentiable function $f(x) = x^3 - 2x + 1$ at $a = 2$. Then graph the curve and this tangent line on the same axes.

1. Define and assign the function to f.
   ```
   > f := x -> x^3-2*x+1;
   ```
2. Right-click and choose *Differentiate*. Then use a label (or right-click again) to assign this new function to df.
   ```
   > df := (Insert label);
   ```
3. Find the tangent line $y = f'(2)(x - 2) + f(2)$ and assign it to L.
   ```
   > L := df(2)*( x-2 ) + f(2);
   ```
4. Plot the function and the tangent line using different linestyles.
   ```
   > plot([f(x),L], x=-2..3, linestyle=[solid,dash], legend=[f(x),L], title=''My Graph'');
   ```

Assignment

Exercises 28, 30, and 34 on page 151 of Stewart. Remember to use the limit definition for each of these. Exercises 6 and 8 on page 150 of Stewart and Exercise 7 on page 65 of *Calclabs*. You do not need to use the limit definition of the derivative for these, but remember to give each of your graphs a title and a legend.