Math 141 Worksheet 7

Show all work for full credit.

1. The graph of \( y = f(x) \) is given. Are the following quantities positive, negative, or zero?
   a. \( f(A) \)  
   b. \( f'(A) \)  
   c. \( f''(A) \)  
   d. \( f(B) \)  
   e. \( f'(B) \)  
   f. \( f''(B) \)  
   g. \( f(C) \)  
   h. \( f'(C) \)  
   i. \( f''(C) \)  
   j. \( f(D) \)  
   k. \( f'(D) \)  
   l. \( f''(D) \)  

2. Sketch the graph of a function \( F \) that has the following properties:
   - \( F \) is everywhere continuous
   - \( F(-2) = 2 \)
   - \( F(0) = 0 \)
   - \( F(3) = -4 \)
   - \( F'(-2) = 0 \)
   - \( F'(3) = 0 \)
   - \( F'(x) > 0 \) for \( x < -2 \)
   - \( F'(x) < 0 \) for \(-2 < x < 3 \)
   - \( F'(x) > 0 \) for \( x > 3 \)
   - \( F''(x) < 0 \) for \( x < 0 \)
   - \( F''(x) > 0 \) for \( 0 < x < 6 \)
   - \( F''(x) < 0 \) for \( x > 6 \)
3. Find the intervals over which \( f(x) \) is increasing, decreasing, concave up, and concave down. Give your answers in interval notation.

\[
f(x) = x^3 - 15x^2 + 72x - 20
\]

4. Given the following graph of \( f'(x) \), determine the intervals over which \( f(x) \) is increasing, decreasing, concave up, and concave down.
5. Determine the absolute maximum and absolute minimum of \( f(x) \) over the given interval.

\[
f(x) = x^3 - 3x^2 - 24x + 20 \quad \text{over } [-5, 5]
\]

6. Determine the absolute maximum and absolute minimum of \( f(x) \) over the given interval.

\[
f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \quad \text{over } [-2, 3]
\]
7. Find the absolute extrema of $f(x)$ over the entire real number line.

$$f(x) = 8x^2 - x^4 + 24$$