This test is designed to give an example of what types of questions may be on the test. Show all work for full credit.

1. Find the absolute extrema of $f(x)$ over the given interval.
   \[ f(x) = 2x^3 - 3x^2 - 12x + 1 \text{ over } [-2, 3] \]

2. A covered box is to be made from a rectangular sheet of cardboard measuring 5 feet by 8 feet. This is done by cutting out the shaded regions of the figure and then folding on the dotted lines. What are the dimensions $x$, $y$, and $z$ that maximize the volume?
3. A page of a book is to contain 27 square inches of print. If the margins at the top, bottom, and one side are 2 inches and the margin at the other side is 1 inch, what size page would use the least paper?

4. A rectangle is to be inscribed in a semicircle of radius 4, as shown below. What are the dimensions of the rectangle if its area is to be maximized?
5. Verify the hypotheses of the Mean Value Theorem are satisfied on the given interval, and find all values $c$ in that interval that satisfy the conclusion of the theorem.

$$f(x) = x^2 + 13x - 30 \text{ over } [2,10]$$

6. The function $s(t)$ describes the position of a particle moving along a coordinate line, where $s$ is in feet and $t$ is time in seconds.

$$s(t) = t^3 - 13.5t^2 + 54t + 5$$

a. Analyze the motion of the particle for $0 \leq t \leq 10$: When is the particle moving to the left? to the right? When is the particle stopped?

b. When is the particle speeding up? slowing down?
7. Evaluate the integral.
\[
\int \frac{e^{2x}}{1 + e^{2x}} \, dx
\]

8. Evaluate the integral.
\[
\int \sec^2(3x) \tan^4(3x) \, dx
\]

9. Evaluate the integral.
\[
\int \frac{(x^3 + 1)^2}{3x^2} \, dx
\]
10. Evaluate the integral.
\[ \int x^3(x^2+5) \, dx \]

11. Evaluate the integral.
\[ \int x \cos(3x^2) \sin(3x^2) \, dx \]

12. Evaluate the integral.
\[ \int \sec^3(5x) \tan(5x) \, dx \]
13. Evaluate the integral.
\[ \int_{0}^{\pi/4} \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} \, dx \]

14. Evaluate the integral.
\[ \int_{0}^{\pi/3} \sec x (\sec x + \tan x) \, dx \]

15. Evaluate the integral.
\[ \int \frac{\sin x}{1 + \cos x} \, dx \]
16. Evaluate the integral.

\[ \int_1^2 \frac{1}{2x + 1} \, dx \]

17. Evaluate the integral.

\[ \int_{-1}^2 \frac{2x^2 - 8}{4x - 8} \, dx \]

18. Evaluate the integral.

\[ \int_0^1 \frac{x + 2}{(x^2 + 4x + 1)^2} \, dx \]
19. Evaluate the integral.
\[ \int \left( x^2 + 1 + \frac{1}{x^2 + 1} \right) dx \]

20. Evaluate the integral.
\[ \int \frac{\cos(\pi/x)}{x^2} \, dx \]

21. Evaluate the integral.
\[ \int (x - 3)^{15} (x + 7) \, dx \]
22. Evaluate the integral. \[ \int \tan(5x) \, dx \]

23. Evaluate the integral. \[ \int_0^5 |2x - 6| \, dx \]

24. Evaluate the integral. \[ \int \frac{1 + x}{1 + x^2} \, dx \]
25. Evaluate the integral.
\[ \int \tan x \ln(\cos x) \, dx \]

26. Evaluate the integral.
\[ \int_{0}^{\pi/2} (4x + \sin x) \, dx \]

27. Solve the following initial-value problem.
\[ \frac{dy}{dx} = 3x^2 + 5 \quad y = 1 \text{ when } x = 1 \]
28. A ball is thrown upward with a speed of 88 ft/sec from the top of a 100 foot tall building. Assuming free-fall motion, what is the maximum height of the ball?

29. Your roommate drops your math book from a dormitory window and it hits the ground in 3 seconds. How high up is the window?

30. A particle is moving so that its velocity is

\[ v(t) = 3 + 2t - t^2 \text{ for } 1 \leq t \leq 4. \]

a. Find the displacement (change in position) of the particle.

b. Find the total distance traveled by the particle.