MATHEMATICS 141 PRETEST 2

This test is designed to give an example of what types of questions may be on the test. Show all work for full credit.

1. Find the equation of the tangent line to the graph of

\[ f(x) = 3x^2 + 7x - 5 \] when \( x = 2 \).

Give your answer in slope-intercept form.

2. Find the derivative \( f'(x) \).

\[ f(x) = \frac{2}{3x^5} + \frac{7}{x} + 6\sqrt{x} + \pi^2 \]

3. Find the derivative \( f'(x) \).

\[ f(x) = \csc x \sec(5x) \]

4. Find the derivative \( f'(x) \).

\[ f(x) = (x + \frac{1}{\sqrt{x}}) \left( \frac{3x + 1}{x^2 - 1} \right) \]
5. Find the derivative $f'(x)$.

$$f(x) = \sin(4x) \cot(3x)$$

6. Find the derivative $f'(x)$.

$$f(x) = \left(\frac{x^2 + 1}{3x^3 + x}\right)^4$$

7. Find the derivative $f'(x)$.

$$f(x) = \sin(\sec(\cos(x^2)))$$

8. Find the derivative $f'(x)$.

$$f(x) = \sqrt{\cos^3(4x) + \sin(x^2)}$$
9. Find the derivative $f'(x)$.

$$f(x) = \sqrt[4]{\ln(x^2)} + 5^x$$

10. Find the derivative $f'(x)$.

$$f(x) = \ln(\cos^2(3x))$$

11. Find the derivative $f'(x)$.

$$f(x) = xe^{\sin(3x)}$$

12. Find the derivative $f'(x)$.

$$f(x) = 2^x \tan(4x) + \ln(8x^3 + 5x^2 + 1)$$
13. Find the derivative $f'(x)$.

$$f(x) = \frac{1}{\cos^2(3x)} + \ln 5$$

14. Find the derivative $f'(x)$.

$$f(x) = x \sin^{-1}(5x^2)$$

15. Find the derivative $f'(x)$.

$$f(x) = \tan^{-1}(\sin^2(x^2))$$

16. Find the derivative $f'(x)$.

$$\frac{e^{3x}(x^3 + 5)^4}{(7x^2 + x - 2)^6}$$
17. Find the derivative $f'(x)$ using logarithmic differentiation.

$$f(x) = (3x^2 + \cos x)^{\tan(x^2)}$$

18. Find $\frac{dy}{dx}$ if the following equation holds.

$$ye^x - x^2 \sin(xy) = 4$$
19. The area of a circular doggie puddle is increasing at a rate of 12 cm²/sec. How fast is the radius increasing at the instant when it equals 10 cm?

20. A 20 meter ladder rests vertically against the side of a barn. A pig that has been attached to the ladder starts to pull the base of the ladder away from the wall at a constant rate of .4 meters per second. Find the rate of change of the top of the ladder after 30 seconds.
21. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m³/min, find the rate at which the water level is rising when the water is 3 m deep.
(Volume of a cone: \( V = \frac{1}{3}\pi r^2 h \).)

22. Evaluate the limit.

\[ \lim_{x \to 0} \frac{4x - \sin(4x)}{x^3} \]
23. Evaluate the limit.

\[ \lim_{x \to 1} \left( \frac{1}{\ln x} - \frac{1}{x - 1} \right) \]

24. Evaluate the limit.

\[ \lim_{x \to \infty} \left( 1 + \frac{2}{x} \right)^x \]

25. Fill in the blank with positive, negative, increasing, decreasing, concave up, concave down, or unknown.

If \( f'(x) \) is increasing then \( f(x) \) is ____________________.

If \( f(x) \) is increasing then \( f'(x) \) is ____________________.

If \( f''(x) \) is negative then \( f'(x) \) is ____________________.

If \( f'(x) \) is decreasing then \( f''(x) \) is ____________________.
26. Find the intervals over which $f(x)$ is increasing, decreasing, concave up, and concave down. Give your answers in interval notation.

$$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x - 5$$

27. Find the critical points of $f(x)$. Use the first derivative test to identify each as the location of a local maximum, local minimum, or neither.

$$f(x) = \frac{3}{2}x^4 + \frac{11}{3}x^3 - 5x^2 + 3$$
28. Given the following graph of $f'(x)$, determine the intervals over which $f(x)$ is increasing, decreasing, concave up, and concave down.

Where would the graph of $f(x)$ have a maximum? ____________

Where would the graph of $f(x)$ have inflection points? ____________

29. The graph of $y = f(x)$ is given. Are the following quantities positive, negative, or zero?

a. $f(A)$
b. $f'(A)$
c. $f''(A)$
d. $f(B)$
e. $f'(B)$
f. $f''(B)$
g. $f(C)$
h. $f'(C)$
i. $f''(C)$
j. $f(D)$
k. $f'(D)$
l. $f''(D)$

[Graphs of $f(x)$ and $f'(x)$ are shown with points labeled A, B, C, and D.]