Math 122 Review 1

1. Complete the table of values for the linear function.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>20</td>
<td>50</td>
<td>80</td>
<td>110</td>
<td>140</td>
</tr>
</tbody>
</table>

Write the equation of this linear function.

\[ m = \frac{50 - 20}{6 - 3} = 10 \]
\[ y - 20 = 10(x - 3) \]
\[ y = 10x - 30 \]
\[ y = 10x - 10 \]

2. Complete the table of values for the exponential function.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>20</td>
<td>50</td>
<td>125</td>
<td>312.5</td>
<td>781.25</td>
</tr>
</tbody>
</table>

Write the equation of this exponential function.

\[ P = P_0 e^{kt} \]
\[ 20 = P_0 e^{-0.305(3)} \]
\[ 50 = P_0 e^{-0.305(6)} \]
\[ \frac{50}{20} = \frac{P_0 e^{-0.305(6)}}{P_0 e^{-0.305(3)}} = e^{3k} \]
\[ 2.5 = e^{3k} \]
\[ \ln 2.5 = 3k \]
\[ \ln 2.5 = 3k \]
\[ k = 0.305 \text{ is stored} \]

\[ P_0 = 8 \]
\[ P = 8 e^{0.305t} \]

3. My swimming pool currently holds 4500 gallons of water, but it is leaking. Find an equation to represent the volume, \( V \), of the pool after \( t \) hours if the decrease is

a. 8% per hour: exponential
\[ V = 4500(1 - 0.08)^t \]

b. 450 gallons each hour: linear
\[ V = 4500 - 450t \]
4. Find the equation of the linear function that passes through the points (6, 250) and (8, 625).

\[ m = \frac{625 - 250}{8 - 6} = 187.5 \]

\[ y - 250 = 187.5(x - 6) \]

\[ y = 187.5x - 1125 \]

\[ y = 187.5x - 875 \]

5. Find the equation of the exponential function that passes through the points (6, 250) and (8, 625).

\[ P = P_0 e^{kt} \]

\[ 250 = P_0 e^{k(6)} \]

\[ 625 = P_0 e^{k(8)} \]

\[ \frac{625}{250} = \frac{P_0 e^{k(8)}}{P_0 e^{k(6)}} \]

\[ 2.5 = e^{2k} \]

\[ \ln 2.5 = \ln e^{2k} \]

\[ \ln 2.5 = 2k \]

\[ 2.5 = e^{2k} \]

\[ P_0 = 160 \]

\[ P = 160 (e^{0.458t}) \]

\[ K = 0.458 \quad \text{Store 1} \]

6. Jason leaves Detroit at 2:00PM and drives at a constant speed west along I-96. He passes Ann Arbor, 40 miles from Detroit at 2:50PM. Express the distance traveled in terms of the time elapsed (in minutes). What speed (in mph) was he traveling?

Detroit \( (0,0) \) \quad Ann Arbor \( (50,40) \)

\[ m = \frac{\Delta D}{\Delta t} = \frac{40-0}{50-0} = \frac{4}{5} \]

\[ D - 0 = \frac{4}{5} (t - 0) \]

\[ D = \frac{4}{5} t \]

\[
\begin{align*}
40 \text{ miles} & , \quad 60 \text{ minutes} \\
50 \text{ minutes} & , \quad 1 \text{ hour}
\end{align*}
\]

\[ = 48 \text{ mph} \]
7. The manager of a furniture factory finds that it costs $2200 to manufacture 100 chairs in one day and $4800 to produce 300 chairs in one day.
   a. Express the cost as a function of the number of chairs produced, assuming it is linear.

   \[ m = \frac{\Delta C}{\Delta q} = \frac{4800 - 2200}{300 - 100} = 13 \text{ \$/chair} \]

   \[ C = 2200 + 13(q - 100) \]

   \[ C = 13q + 900 \]

   b. What is the vertical intercept of the graph and what does it represent?

   \[(0, y) \quad C = 13(0) + 900 \]

   \[ C = 900 \quad \text{V-Intercept } (0, 900) \]

   The company will have to pay $900 in fixed cost (when they have not yet produced any chairs).

8. A company that manufactures small canoes has a fixed cost of $1800. The company can produce 50 canoes for a total cost of $2800. The total revenue gained by selling 50 canoes is $4000.
   a. Write the cost function, \( C(q) \).

   \[(0, 1800) \quad m = \frac{2800 - 1800}{50 - 0} = 20 \text{ \$/canoe} \]

   \[ C = 20q + 1800 \]

   b. Write the revenue function, \( R(q) \).

   \[ R = pq \]

   \[ 4000 = p(50) \]

   \[ p = 80 \]

   \[ R = 80q \]

   c. Determine the break-even point. Describe what this means.

   \[ 80q = 20q + 1800 \]

   \[ 60q = 1800 \]

   \[ q = 30 \text{ Canoes} \]

   Break-even Point

   At a production level of 300 canoes, the company spends exactly what it earns in revenue, so profit is $0. The company needs to sell more than 30 canoes to make profit.
9. I am choosing between two long-distance telephone plans. Plan A has a monthly fee of $20 with a charge of $0.05 per minute. Plan B has a monthly fee of $5 with a charge of $0.10 per minute. How should I determine which plan is best for me? Be specific with your answer.

\[ C_A = 20 + 0.05x \]
\[ C_B = 5 + 0.10x \]
\[ 20 + 0.05x = 5 + 0.10x \]
\[ 15 = 0.05x \]
\[ x = 300 \text{ minutes} \]

If you talk more than 300 minutes each month, choose Plan A.
If you talk less than 300 minutes each month, choose Plan B.
If you talk exactly 300 minutes, the choice does not matter.

10. The population models for several towns with time \( t \) in years are given below:

<table>
<thead>
<tr>
<th>Town</th>
<th>Population Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( P = 1200(1.03)^t )</td>
</tr>
<tr>
<td>B</td>
<td>( P = 900(1.14)^t )</td>
</tr>
<tr>
<td>C</td>
<td>( P = 1000(0.95)^t )</td>
</tr>
</tbody>
</table>

a. Which town’s population is growing the fastest? What is the percent growth rate?

Town B
\[ a = 1 + r \]
\[ 1.14 = 1 + r \]
\[ r = 0.14 \rightarrow 14\% \text{ growth} \]

b. Are any of the towns decreasing in size?

Yes, Town C, we can tell because \( a = 0.95 < 1 \).

c. Which town has the largest initial population?

Town A \( P_0 = 1200 \) people.

11. What interest rate, compounded continuously, is equivalent to an 8% rate compounded annually?

\[ a = e^k \]
\[ a = 1 + r \]
\[ 1.08 = e^k \]
\[ a = 1 + 0.08 \]
\[ \ln 1.08 = k \ln e^k \]
\[ k = 0.076916 \rightarrow 7.6916\% \]
12. Traces of burned wood along with ancient stone tools in an archaeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, approximately how long ago was the tree cut and burned?

Half-life is 5600 years

\[ 0.5 = e^{-\frac{5600}{k}} \]

\[ \ln 0.5 = \ln e^{-\frac{5600}{k}} \]

\[ \ln 0.5 = \frac{5600}{k} \]

\[ k = -\frac{\ln 0.5}{5600} \]

Let \( p_0 = 100 \) so \( 0.0167 = e^{-0.000124t} \)

\[ \ln 0.0167 = \ln e^{-0.000124t} \]

\[ \ln 0.0167 = -0.000124t \]

\[ t = \frac{\ln 0.0167}{-0.000124} \]

\[ t = 33062.445 \text{ years ago.} \]

13. A small town’s population is modeled by the following equation

\[ P(t) = 5000(1.03)^t \text{ people,} \]

where \( t \) represents the number of years since 1998.

a. Find the average rate of change of \( P(t) \) between \( t = 7 \) and \( t = 12 \). Give units.

\[ \frac{P(12) - P(7)}{12 - 7} = \frac{7128.804 - 6149.369}{5} = 195.887 \text{ people/year} \]

b. Interpret your answer from (a) in everyday language.

The small town’s population increased by an average of 195.887 people each year between 2005 and 2010.

14. A clean up of a polluted lake will remove 5% of the remaining contaminants every year, beginning in 2006. The goal is to reduce the quantity of contaminants to \( \frac{1}{10} \) of its present level. When will this be achieved?

\[ a = 1 - 0.05 = 0.95 \]

Let \( p_0 = 100 \) so \( P = 100(0.95)^t \)

where \( t \) is years since 2006.

\[ 2006 + 44.891 = 2050.891 \]

The clean-up will be completed close to the end of 2050.\]

\[ \frac{1}{10} (100) = 10 \]

\[ 10 = 100(0.95)^t \]

\[ 0.10 = 0.95^t \]

\[ \ln 0.10 = \ln 0.95^t \]

\[ \ln 0.10 = t \ln 0.95 \]

\[ t = \frac{\ln 0.10}{\ln 0.95} \]

\[ t = 44.891 \text{ years} \]
15. The height of an object in feet above the ground is given in the following table:

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y ) (feet)</td>
<td>10</td>
<td>45</td>
<td>70</td>
<td>85</td>
<td>90</td>
<td>85</td>
<td>70</td>
</tr>
</tbody>
</table>

Compute the average velocity over the interval \( 0 \leq t \leq 3 \). Give units.

\[
\frac{85-10}{3-0} = \boxed{25 \text{ ft/s}}
\]

16. The speed that a car can achieve in 10 seconds is inversely proportional to its weight. (That is, the more the car weighs, the slower it will be going.) After 10 seconds, a car that weighs 2400 pounds can achieve a speed of 44 miles per hour. If the car weighed 1600 pounds, how fast would it be going?

\[
S = K \left( \frac{1}{W} \right)
\]

44 = \( K \left( \frac{1}{2400} \right) \)

\[
K = 105600
\]

\[
S = 105600 \left( \frac{1}{1600} \right)
\]

\[
S = \boxed{66 \text{ mph}}
\]

17. Suppose \( S(q) \) is the price per unit (in dollars) of a certain good at which producers will supply \( q \) goods to the market, and suppose that \( D(q) \) is the price per unit (in dollars) at which consumers will buy \( q \) goods.

a. Which price is higher, \( p_1 = D(100) \) or \( p_2 = D(150) \)? Explain.

Consumers demand less when the price is high, so \( p_1 \), the price when consumers buy 100 items, must be higher than \( p_2 \), the price when consumers buy 150 items.

b. Which price is higher, \( p_1 = S(100) \) or \( p_2 = S(150) \)? Explain.

Manufacturers supply more when the price is high, so \( p_2 \), the price when manufacturers supply 150 items, must be higher than \( p_2 \), the price when manufacturers supply 100 items.

c. If \( D(150) = 8 \) and \( S(100) = 8 \), what will you predict about the future selling price of the good (currently at $8)? Justify your prediction.

Currently, consumers want 150 items but manufacturers are only supplying 100 items, so we have a shortage and the future selling price will be higher.