Derivatives Basics

Differentiation Rules

Let $y = f(x)$ and $y = g(x)$ be functions which are differentiable at x. Let a and b be constants.

Linearity:

$$D_x\left[af(x) + bg(x)\right] = af'(x) + bg'(x)$$

Product Rule:

$$D_x\left[f(x) \cdot g(x)\right] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Quotient Rule:

In the case $g(x) \neq 0$

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left[g(x)\right]^2}$$

Chain Rule:

In the case that f is diferentiable at x and g is differentiable at $f(x)$

$$D_x\left[g\Big(f(x)\Big)\right] = g'\Big(f(x)\Big)f'(x)$$

Main Topic # 1: [Identification] The first step of applying the product rule is knowing when to apply the rule. For the following identify at least one way you can view this as the multiplication of two functions.

i. $f\left(t\right) = \left(4t^2 - t\right)\left(t^3 - 8t^2 + 12\right)$

ii. $y = \left(1 + \sqrt{x^3}\right)\left(x^{-3} - 2\sqrt[3]{x}\right)$

iii. $h\left(z\right) = \left(1 + 2z + 3z^2\right)\left(5z + 8z^2 - z^3\right)$

iv. $g\left(x\right) = x^3$

v. $R\left(w\right) = \dfrac{3w + w^4}{2w^2 + 1}$

vi. $f\left(x\right) = 4x^3\sqrt{x}$

Main Topic # 2: [Your First Derivative] The first derivative to learn (and I'm sure you have seen the 'proof' of this in your class, and if you have not yet seen it don't worry you will) is the derivative of x and the derivative of a constant.

Derivative of x

The derivative of a constant is the most basic of all derivatives, you may recall that a derivative is a function whose output is how much the original function is changing at that time. An example from real life is if $f(t)$ is a function describing your how far you have traveled on a trip at time t then... somewhere then $f'(t)$ is your speed (well velocity but you get the idea) at time t. If we understand this idea then we see immediately for some number $c \in \mathbb{R}$

$$D_x\Big[c\Big] = 0$$

The derivative of the 'identity function' $f(x) = x$ is another one of the most basic derivatives.

$$D_x\Bigg[x\Bigg] = 1$$

Warning: You may have seen D_x written as $\dfrac{d}{dx}$

Using the *linearity* property from above we can then take the derivative of more complicated functions like

Example:

$$\frac{d}{dx}\Bigg[2x+7\Bigg] = 2\frac{d}{dx}\Bigg[x\Bigg] + \frac{d}{dx}\Bigg[7\Bigg] = 2 \cdot 1 + 0 = 2$$

Now using the Product rule we can find some awesome derivatives.

Problem 1. Use the **Product Rule** and **Linearity** to find the following derivatives (you may have been given rules for these but try **not** to use those rules just use the product rule to try and get practice!)

i. $\dfrac{d}{dx}(x^2)$

ii. $\dfrac{d}{dx}(x^3)$

iii. $\dfrac{d}{dx}(x^4)$

iv. $\dfrac{d}{dx}(3x^2+3)$

viii. $\dfrac{d}{dx}\left(1+x^3\right)\left(x^{-3}-2x\right)$

v. $\dfrac{d}{dx}\left(3x^2+3\right)\cdot\left(4x^2-4\right)$

ix. $\dfrac{d}{dz}\left(1+2z+3z^2\right)\left(5z+8z^2-z^3\right)$

vi. $\dfrac{d}{dx}(3x^2)\cdot(4x^3-4x+7)$

x. $\dfrac{d}{dx}(4x^3-1)\cdot(x+1)$

vii. $\dfrac{d}{dt}\left(4t^2-t\right)\left(t^3-8t^2+12\right)$

xi. $\dfrac{d}{dx}(x-1)\cdot(x+1)$