**The Limits that come up in Improper Integrals**

**Main Topic # 1:** ['Basic' Limits] These are some of the most used limits of sequences used Calc II.

**Commonly Occurring Limits**

- $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$
- $\lim_{n\to\infty} c^{\frac{1}{n}} = 1 \quad (c > 0)$
- $\lim_{n\to\infty} c^n = 0 \quad (|c| < 1)$
- $\lim_{n\to\infty} \left(1 + \frac{c}{n}\right) = e^c \quad$ (any $c$)
- $\lim_{n\to\infty} \sqrt[n]{n} = 1$
- $\lim_{n\to\infty} \frac{c^n}{n!} = 0 \quad$ (any $c$)

**Basic Properties**

If $\{a_n\}$ and $\{b_n\}$ are both convergent sequences then,

- $\lim_{n\to\infty} (a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$
- $\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$
- $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$ provided $\lim_{n\to\infty} b_n \neq 0$
- $\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$
- $\lim_{n\to\infty} (a_n\, b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right)$
- $\lim_{n\to\infty} a_n^p = \left[\lim_{n\to\infty} a_n\right]^p$ provided $a_n \geq 0$

**Squeeze for Sequences**

If $a_n \leq c_n \leq b_n$ for all $n > N$ for some $N$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L$ then $\lim_{n\to\infty} c_n = L$.

**Absolutely Zero**

If $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$.

**Problem 1.** Determine the following limits are convergent or divergent, if they are convergent find the limit.

i. $\left\{\frac{3n^2 - 1}{10n + 5n^2}\right\}_{n=2}^{\infty}$

ii. $\left\{\frac{\mathbf{e}^{2n}}{n}\right\}_{n=1}^{\infty}$

iii. $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty}$

iv. $\{(-1)^n\}_{n=0}^{\infty}$

**Problem 2.** Determine the following limits are convergent or divergent, if they are convergent find the limit.

i. $\left\{ \dfrac{n^2 - 7n + 3}{1 + 10n - 4n^2} \right\}_{n=3}^{\infty}$

ii. $\left\{ \dfrac{(-1)^{n-2} n^2}{4 + n^3} \right\}_{n=0}^{\infty}$

iii. $\left\{ \dfrac{\mathbf{e}^{5n}}{3 - \mathbf{e}^{2n}} \right\}_{n=1}^{\infty}$

iv. $\left\{ \dfrac{\ln(n+2)}{\ln(1+4n)} \right\}_{n=1}^{\infty}$

v. $\left\{ \dfrac{5 + n^3}{2n^2 - 8n + 1} \right\}_{n=0}^{\infty}$

vi. $\left\{ \dfrac{6n^4 + 9n^2}{9n^4 - 8n^2 + 7} \right\}_{n=11}^{\infty}$

vii. $\left\{ \dfrac{(-1)^{n+7}(2 - 8n)}{n^2 + 9} \right\}_{n=2}^{\infty}$

viii. $\{\cos(n\pi)\}_{n=0}^{\infty}$

ix. $\left\{ \dfrac{n+1}{\ln(6n)} \right\}_{n=2}^{\infty}$

x. $\left\{ \cos\left( \dfrac{3}{n+1} \right) \right\}_{n=1}^{\infty}$