The Limits that come up in the ratio test

Main Topic # 1: [‘Basic’ Limits] These are some of the most used limits of sequences used Calc II.

Commonly Occurring Limits

- $\lim\limits_{n\to\infty} \frac{\ln(n)}{n} = 0$
- $\lim\limits_{n\to\infty} c^{\frac{1}{n}} = 1 \quad (c > 0)$
- $\lim\limits_{n\to\infty} c^n = 0 \quad (|c| < 1)$
- $\lim\limits_{n\to\infty} \left(1 + \frac{c}{n}\right) = e^c \quad (\text{any } c)$
- $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$
- $\lim\limits_{n\to\infty} \frac{c^n}{n!} = 0 \quad (\text{any } c)$

Basic Properties

If $\{a_n\}$ and $\{b_n\}$ are both convergent sequences then,

- $\lim\limits_{n\to\infty} (a_n \pm b_n) = \lim\limits_{n\to\infty} a_n \pm \lim\limits_{n\to\infty} b_n$
- $\lim\limits_{n\to\infty} ca_n = c \lim\limits_{n\to\infty} a_n$
- $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n}$ provided $\lim\limits_{n\to\infty} b_n \neq 0$
- $\lim\limits_{n\to\infty} ca_n = c \lim\limits_{n\to\infty} a_n$
- $\lim\limits_{n\to\infty} (a_n\, b_n) = \left(\lim\limits_{n\to\infty} a_n\right)\left(\lim\limits_{n\to\infty} b_n\right)$
- $\lim\limits_{n\to\infty} a_n^p = \left[\lim\limits_{n\to\infty} a_n\right]^p$ provided $a_n \geq 0$

Squeeze for Sequences

If $a_n \leq c_n \leq b_n$ for all $n > N$ for some N and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} b_n = L$ then $\lim\limits_{n\to\infty} c_n = L$.

Absolutely Zero

If $\lim\limits_{n\to\infty} |a_n| = 0$ then $\lim\limits_{n\to\infty} a_n = 0$.

Ratio Test for Series

Instead of recalling all of the ratio test lets only recall the set up!

Suppose we have the series $\sum a_n$. Define,

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

Problem 1. For each of the following find L defined in the above box!

i. $\displaystyle\sum_{n=0}^{\infty} \frac{n^3+n^2}{(n+1)!}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{5^{1-n}(n+1)}$

iii. $\displaystyle\sum_{n=0}^{\infty} \frac{(2n-1)!}{(3n)!}$

iv. $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{4+n}}{3n^2+1}$

v. $\displaystyle\sum_{n=2}^{\infty} \frac{4^{1+\frac{1}{2}n}n^2}{3^{2+n}(n+3)}$

vi. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{(-1)^{n+2}(n^2+n+1)}$

vii. $\displaystyle\sum_{n=3}^{\infty} \frac{6^{-2n}(n-4)}{4^{3-2n}(2-n^2)}$

viii. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n(n+1)}{n^2+1}$

ix. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{1-2n}}{n^2+1}$

x. $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}}$

xi. $\displaystyle\sum_{n=3}^{\infty} \frac{\mathbf{e}^{4n}}{(n-2)!}$

xii. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{6n+7}$