1.2 Linear Functions!

- it's now time to say the "Big" word of calculus

CHANGE

[open up for discussion]

Q: {how can we tell if something has changed?}

A: it's different...

→ so what's the difference (subtraction)

we call this change in p

$$\Delta p = p_1 - p_0$$

some other value...

Recall we call this initial

So what? (e.g.) P(t) = price of a stock w/ respect to time

take two prices a stock has been

$\$1 = P_0$ $\quad P_1 = \$500$

$\Delta p = 500 - 1 = 499$ wow that's a big change

- yet what times gave me those values

what if it was $t = 1983$ and $t = 5001$

so $\Delta t = 5001 - 1983 = 3018$

and check out!

we call this Relative change! (it's the change of P relative to the change of t)

$$\frac{\Delta p}{\Delta t} = \frac{499}{3018} \approx 0.16 \leftarrow \text{not very wow!}$$!!

in general

$$\frac{\Delta \text{ dep. Var.}}{\Delta \text{ indep. Var.}} = \text{relative change}$$

→ Constant relative change

(big deal!)

• what if the relative change was constant

→ then grab a point (t_0, p_0) {on the graph} — an actual point

then for <u>any</u> other point (t, p) {on the graph}

we must have

constant no matter how I fill in (t, p) on the graph

$$\frac{p - p_0}{t - t_0} = c$$

So

$$p - p_0 = c(t - t_0)$$

$$\Rightarrow p = \underset{\#}{c}\,t + \underbrace{(p_0 - ct_0)}_{\#}$$

i.e.

$$p(t) = ct + \#$$

i.e.

$$p(t) = mt + b \quad \text{A Line!}$$

Constant relative change Means Line

(huge deal!)

$\boxed{ex}$ So the price of a stock is dependent on time

and at time 0 the price is \$2

and at the time 1 the price is \$1

[If we know that the price as a function of time has constant relative change can we "write" out the function?]

So on the graph is the points $(0,2)$ and $(1,1)$

So

$$\frac{\Delta p}{\Delta t} = \frac{1-2}{1-0} = -1$$

and before we saw

$$ct + (p_0 - ct_0)$$

$$p(t) = (-1)t + (2 - (-1)(0))$$

$$= -t + 2$$

<u>Wow!</u>

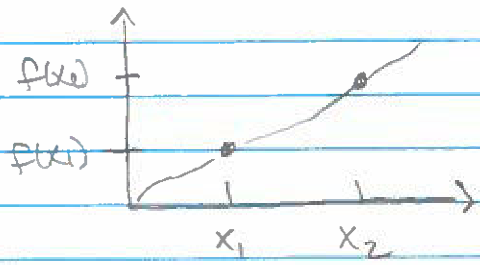
★ Recall 5.1) one number line ... & and w... it means for subtract...

★ Recall ... {continuous curve}

Increasing & decreasing fncts:

defn: a function is increasing...
whenever
$x_1 < x_2$ then $f(x_1) < f(x_2)$

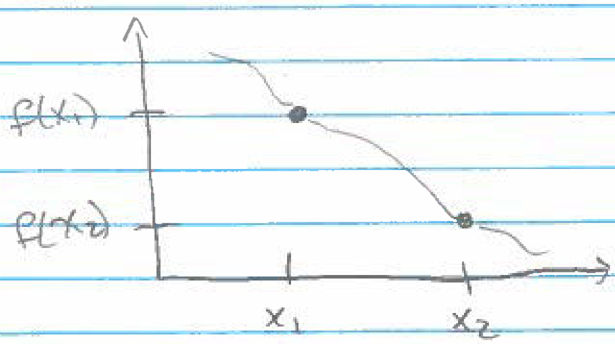
what does that look like:



★ Rob remember to use colors!

defn: a function is decreasing
whenever
$x_1 < x_2$ then $f(x_1) > f(x_2)$

what does that look like:



what does
(Avg) Rate of Change

have to say about increasing & decreasi

Increasing:

$x_1 < x_2$ means $x_2 - x_1 > 0$ {posi

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{\underset{+}{x_2 - x_1}} > 0$$

* {positive} *

increasing:
$f(x_1) < f(x_2)$
means
$f(x_2) - f(x_1) > 0$
{positive}
+

Decreasing:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0$$

{negative}

Decreasing:
$f(x_1) > f(x_2)$
means
$f(x_2) - f(x_1) < 0$
{negative}
−

* Remember
this!

very important!

percent change:
(relative)

Book always means this!

{w/ step size 1} → i.e. $\Delta x = 1$

$$\boxed{\frac{\Delta y}{y_0}}$$

e.g. if we increase cost depends on original cost if it's a "Big" change

(a) a gallon of gas costing \$2.25

(b) a cell phone costing \$180

- for both calculate the percent change {step size 1} for an increase of \$2

Boo

$$\frac{2.25 - 2.00}{2.25} = \frac{.25}{2.25} = \frac{1}{9}$$

$$\frac{180 - 2}{180} = \frac{178}{180}$$

Applications to economics:

defn/ The cost function
gives total cost of producing a quantity "q"
of some good

notation: $c(q)$

* first we will assume the cost of a "unit" is constant *

$$\text{Total cost} = \text{fixed costs} + (\text{cost of a unit}) \cdot q$$

quant... go...

a cost that doesn't change

how much it costs to make 1 "unit"

→ employee and raw...

like:
→ rent
→ lights
etc...

e.g. consider a company that makes TV's. The Factory and Machinery needed to begin producing TV's and occurs even when no TV's are made, has a cost of $24,000. The cost of labor and raw materials is $10 per TV.

- what are the "fixed costs", "cost of a unit"
- write the equation for $c(q)$
- graph the equation

defn marginal cost is the (avg.) rate of change
*{speed cost increases/decreases}

• what is the marginal cost of the proceeding example?

* cost function is completely defined by the and fixed costs *

defn Revenue function Total revenue received for selling quantity "q" of a good notation: R(q)

revenue = price • quantity

e.g. Consider the TV example from before

• Say you sell a TV for $200 dollars when do you "Break even" R(q) = 200

* draw the graphs *

* ask the question: "what does "Break ev

Finally do some algebra to solve for intersection

$200q = 2400 + 10q$ * solve for q *

* Remember to use colors *

profit fnct:

$P(q) = R(q) - C$

Supply & demand:

Supply curve:

input

Supply is a function of prize (p)
and the output is how much (q) quantity the producer willing to make @ that prize

demand curve:

input

demand is a function of price (p)
and the output is how much (q) quantity "the mark is willing to purchase @ that price

defn
we say the Market is @ equilibrium when

supply = demand

and call the (p_0, q_0) where these meet

the equilibrium price & quantity

draw a pic

e.g. find the equilibrium price

if supply = 3p − 50
demand = 100 − 2p

3p − 50 = 100 − 2p

Start here!

Recall

exponent

Rules:

$$a^m \cdot a^n = a^{m+n}$$

☆ keep on the board

$$(a^m)^n = a^{m \cdot n}$$

use color!

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

always!

$$\frac{1}{a^n} = a^{-n}$$

Exponential functions:

example:

population of New York

	year	pop. (mil)
t=0	2003	12.853
t=1	2004	13.290
t=2	2005	13.747
t=3	2006	14.225
t=4	2007	14.721
t=5	2008	15.234
t=6	2009	15.757

☆ draw a graph!

let's look @ (avg.) rate of chan

$$\frac{13.290 - 12.853}{1 - 0} = 0.437$$

$\neq$

$$\frac{13.747 - 13.290}{2 - 1} = 0.457$$

So Not a line

since Not constant (avg.) ra

Now look @

relative (percent) change {step size 1}

$$\left.\begin{array}{l} \dfrac{13.290 - 12.853}{12.853} = 0.034 \\[2ex] \dfrac{13.747 - 13.290}{13.290} = 0.034 \\[2ex] \dfrac{14.226 - 13.747}{13.747} = 0.034 \end{array}\right\} \text{all equal}$$

☆ So $\boxed{\text{constant relative (percent) change}}$ ☆ {w/ step siz

[last time w/ constant (avg) rate of change we got a lin

what do we get now?]

So we want a function

$p(t)$ initial population

we just saw: $p(0) = 12{,}853$ and...

$$\frac{p(1)-p(0)}{p(0)} = 0.034$$

$$\Rightarrow \quad p(1)-p(0) = p(0)\cdot(0.034)$$

$$\Rightarrow \quad p(1) = p(0)\cdot(0.034) + p(0)$$

$$\Rightarrow \quad p(1) = p(0)(1+0.034) = p(0)(1.034)$$

and $$\frac{p(2)-p(1)}{p(1)} = 0.034$$

$$\Rightarrow \quad p(2)-p(1) = p(1)(0.034)$$

$$\Rightarrow \quad p(2) = p(1)(0.034) + p(1)$$

$$\Rightarrow \quad p(2) = p(1)(1+0.034) = p(1)\cdot(1.034)$$

$$= p(0)(1.034)(1.034) = p(0)(1.034)^2$$

and $$\frac{p(3)-p(2)}{p(2)} = 0.034$$

$$\Rightarrow \quad p(3)-p(2) = p(2)(0.034)$$

$$\Rightarrow \quad p(3) = p(2)(1.034) = p(0)(1.034)^2(1.034)$$

$$= p(0)(1.034)^3$$

So $$p(t) = p(0)(1.034)^t = 12{,}853(1.034)^t$$

The natural log

euler's # (pronounced oiler's #)

$e = 2.17\ldots\ldots$

Q: So what's the opposite of e^x?

like -3 is the opposite of $+3$

and $\frac{1}{3}$ is the opposite of times 3

Ans: ln {the natural log}

has the "opposite rules as exponents"

$\boxed{\text{Rules of Ln}}$

$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$

$\ln(A \cdot B) = \ln(A) + \ln(B)$

$\ln(A^p) = p\ln(A)$

$\ln(e^x) = x$

$e^{\ln(x)} = x$

$\ln(1) = 0$ since $e^0 = 1$ and $\ln(e) = 1$

* there is no ln of negative #'s or zero!

So $e^{\ln(a)} = a$

So we can write

$$a^t = \left(e^{\ln(a)}\right)^t = e^{t \cdot \ln(a)}$$

So from before

$$P(t) = P_0(1+r)^t = P_0 e^{t \cdot \ln(1+r)}$$

* we call $\ln(1+r)$ the continuous growth/deca

So in general:

growth/decay

$$P(t) = P_0 e^{k \cdot t}$$

initial population

where $k = \ln(1+r)$

where

r is the constant relative change

Compound Intrest

These things may sound foreign to you
but banks use them every day

this is the relative cha

annual compounding intrest
and continuous compounding intrest

this is the continuous rate

eg.) A bank offers 8% intrest on a savings account, would you make more money if they compounded annually or continuously? [say you invested $4000]

annually

$$P(t) = P_0 (1+r)^t = 4000(1.08)^t$$

so $p(1) = 400(1.08) \approx$

Continuously

$$p(t) = P_0 e^{rt} = 4000 e^{0.08t}$$

so

$$p(1) = 4000 e^{0.08} \approx$$

let's see decay

e.g.) when a patient is given a medication the drug enters the blood stream. The rate @ which the drug is metabolized and eliminated depends on the particular drug. For a certain antibiotic approximately 30% of the drug is eliminated every hour. A Typical dose of this medicine is 200 m.g.

So $Q(t)$ the quatity of the drug left in the bloodstream after t-hours of injection

is $Q(t) = Q_0 e^{k \cdot t}$, where $Q_0 = 200 = Q(0)$

and

$$r = \frac{Q(1) - Q(0)}{Q(0)} = -0.3$$

this is decay!

why is it negative?

So $k = \ln(1 - 0.3) = \ldots$

the continuous growth rate

thus

$$Q(t) = Q_0 e^{\ln(1-0.3) \cdot t}$$

example During the 1980's, Costa Rica had the highest deforestation rates in the world at 2.9% per year (this is the rate that land covered by forests is shrinking) Assuming this rate continues what percent of the land in Costa Rica covered by forest in 1980 will still be covered in 2015?

[what are we looking for]
if $f(t)$ is land covered by forests

we are looking for

$$\frac{f(35)}{f(0)}, \text{ well}$$

$$f(t) = f_0 (1-0.029)^t$$

So

$$\frac{f(35)}{f(0)} = (1-0.029)^{35}$$

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example Suppose $f(t)$ is an exponential and $f(20) = 88.2$ and $f(23) = 91.4$
(a) find the continuous growth rate.
(b) when will it equal 100

Sol'n (a) $\frac{91.4}{88.2} = \frac{\cancel{f(0)} e^{k \cdot 23}}{\cancel{f(0)} e^{k \cdot 3}} = e^{3 \cdot k}$

$$\frac{\ln\left(\frac{91.4}{88.2}\right)}{3} = k \approx$$

(b) find $f(0)$, $88.2 = f(0) e^{k \cdot 3}$ so $f(0) = \frac{88.2}{e^{k \cdot 3}}$

$f(0) e^{k \cdot t}$ ... (100)

doubling time & half time

defn: doubling time in exponential growth is the time it takes to double

defn: half-life in exponential decay is the time it takes to cut in half

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example:
(a) if the continuous rate is 0.01 what is the doubling time?

(b) if the continuous rate is -0.01 what is the half life?

Sol'n (a)

$$\frac{2 \cdot P_0}{P_0} = \frac{P_0 e^{0.01 t}}{P_0} \Rightarrow 2 = e^{(0.01)t} \Rightarrow \frac{\ln(2)}{0.01} = t$$

(b)

$$\frac{\frac{1}{2} P_0}{P_0} = \frac{P_0 e^{(-0.01)t}}{P_0} \Rightarrow \frac{1}{2} = e^{(-0.01)t} \Rightarrow \frac{\ln(\frac{1}{2})}{-0.01} =$$

$$\Rightarrow \frac{\ln(2)}{0.01} = t$$

what!!!

they are the same

what!!!??

Group work:

G.W. 1 The consumer price Index (CPI) for a given year is the amount of money in that year has the same purchasing power as \$100 in 1983. At the start of 2009, the CPI was 211. write a formula for the CPI as a function of years after 2009, assuming the CPI increases by 2.8% per year.

Soy bean production: (in millions)

G.W. 2

year	2000	2001	2002	2003	2004
productn	161.0	170.3	180.2	190.7	201.8

→ does this data suggest a line or an exponential function or neither? why?

→ Can you write a function for this data? if so do!

G.W. 3 An air freshener starts w/ 30 grams and evaporates 12% a day. write a formula for this decay. what is the air freshener's half life?

Chapter 1

Secn 1.6

p. ... p21

group work

G.W. 1

A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per h

what's the half life of caffine in the body after drinking a cup of coffee?

G.W. 2

Prozak has a half life of about 3 days. what precentage is in your blood stream after 2 days?

G.W. 3

In 2012 the world's population was 7 and is projected to reach 8 billion by the year 2025. what annual growth rate is projected? what continuous rate is this?

New functions from old:

* A look to the future:

cost = fixed cost + variable cost

$\# \quad + \quad f(q)$

↑ some function of quantity

So take our first example:

$C(q) = b + mq$ { baby example / Base case

then

$C\left(\frac{1}{m}f(q)\right) = b + f(q)$ any other example

* So we can make any cost function "from" the baby example *

$\boxed{\text{Composition:}}$ * don't forget colors *

$f(x) = x^2 + 7x + 2$

$f(2) = (2)^2 + 7(2) + 2$

$f(x+h) = (x+h)^2 + 7(x+h) + 2$ * what $g(x)$

$f(☺) = (☺)^2 + 7(☺) + 2$

$f(g(x)) = (g(x))^2 + 7(g(x)) + 2$

relationship between revenue & demand

{what if Both price and quantity are "variable" }

* com of A:

polynomial, Revenue (/example of composition)

example Rob's "Get Funky ent!" finds that the average # of people attending a dubstep concert is 70 if the price is $50 per person. At a price of $30 per person the average # of people in attendance is 110. Assuming the demand is a line write the revenue as as a function of price and @ what prices would be bad and give you zero revenue

demand: $\frac{110-70}{30-50} = \frac{40}{-20} = -2$ people per dollar

demand → so $-2 = \frac{q-70}{p-50} \Rightarrow -2p+170 = q$

price ↓ , qu ↙

$$R = p \cdot q = p(-2p+170) = -2p^2+170p$$

$$0 = -2p^2+170p$$

$$p = \frac{-170 \pm \sqrt{(170)^2+8}}{-4}$$