Math 122 Lecture 0 Rob Vandermolen

Mathematics is a language! (a language that is suited to describe IDEAS)

our very first Translation: (The whole point of this class)

Calculus = CHANGE!

Before we can completly understand this translation lets explore this language further!

Functions: [Chapter 1.1 in your book]

The name of this course is:

"Calculus for Buisness Administration and Social sciences"

To keep this title true we will have two "running questions" (even some what related..)

B.A: "How much do you charge for bread?"

Soc. Sci.: "what is the population of this town?"

[open up for discussion]

* I believe the most appropriate answer is:

"well... it Depends..." *key concept*

let's use the language of mathetics to say this...
we will use a special "part of speech"
known as Functions!

R.V.

• Functions

let's say we were trying to answer the B.A. question

in math first we "name" our answer

[you may in the past seen letters like f, g, h...]

since our answer is the price of bread let's use the letter

$$P$$

next we would like to indicate how it "depends" on stuff

for example *[open up for discussion]*

price could depend on:

→ Supply
→ Demand
→ Quantity sold

} these two are quite complicated yet for a moment let's pretend they are simplier

* the world is super complicated and to hope to say anything about it we need to simplify a lot! *

to write this dependence we use parenthesis and seperate them w/ commas

↳ we say "of" or "function of" in this example: "price is a function of supply...etc."

$$\text{price} = P(S, D, Q)$$

we call the "price" the Dependent variable since it "depends" on the other variables

we call the stuff that it depends on the Independent variables

* we think of the independent variables as Inputs to our function
  - all possible inputs is called: Domain

  and we think of the dependent variable as the output of our function
  - all possible outputs is called: Range

→ describing a "real world" problem in the language of Mathematics is reffered to as Mathematical Modeling and this function is called a model

Functions

to describe functions/models, we can give some arithmatic description using polynomials, exponentials, ... etc. {which we will do later}
or we can just list the values the function takes given an input

for example we can look at the price of a stock as a function of time...

ex

price = $p(t)$

input independent variable

year [t]	1980	1981	1982	. . . . . .
price [p]	1	1.5	.5	

output dependent variable

---

ex let's see a quick example of the "arithmatic" description, specially a polynomial
{don't worry we will discuss this more later}

$$\text{price} = p(t) = \frac{1}{2}t^2 + 3t$$

when you give an input say 2 you just write 2 every where t appears

like:

finally you add them together

$$p(2) = \frac{1}{2}(2)^2 + 3(2) = 2 + 6 = 8$$

remember order of operations first square 2 then times by $\frac{1}{2}$

this means 3 times 2

• Functions

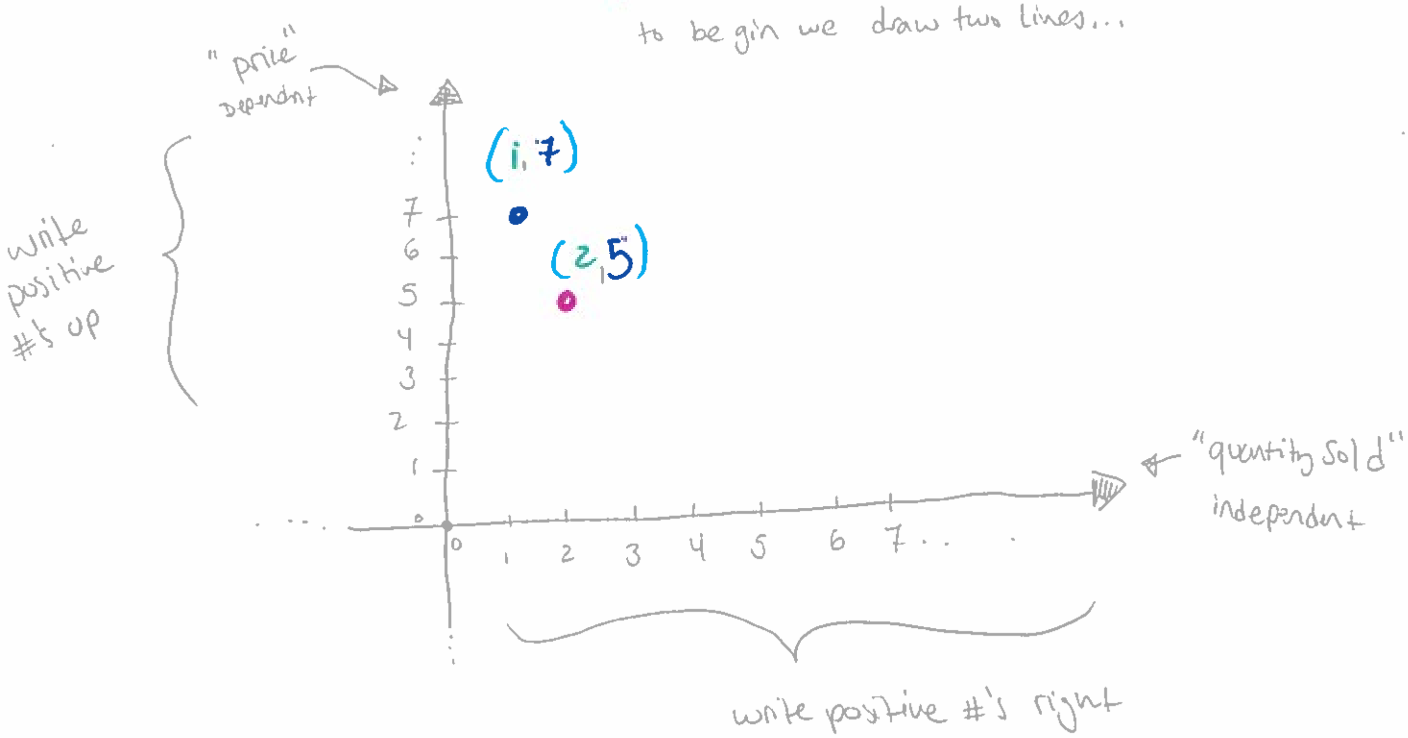
To understand 'functions/models', we often draw a picture to describe it, this picture is called The graph

(R.V.)

ex) let's simplify our first example further, let's pretend that price only depends on the quantity sold, that is

$$\text{price} = P(q)$$

to begin we draw two lines...



• The first point illustrates that if we sell 1 "unit" we sell our "product" for 7 "currency" and we write it's "location" on the picture as

$$(1,7)$$

input output

• The next point illustrates that if we sell 2 "units" we sell our "product" for 5 "currency" and we write it's "location" on the picture as

$$(2,5)$$

* this is an example of discrete

Functions

R.V.

in math we like stuff to make sense and be consitent

So we will reserve the word function for only when given an input we only get one out put, that is for example we will not let

$$p(3) = 2 \text{ and } p(3) = 5$$

it can only have a single output

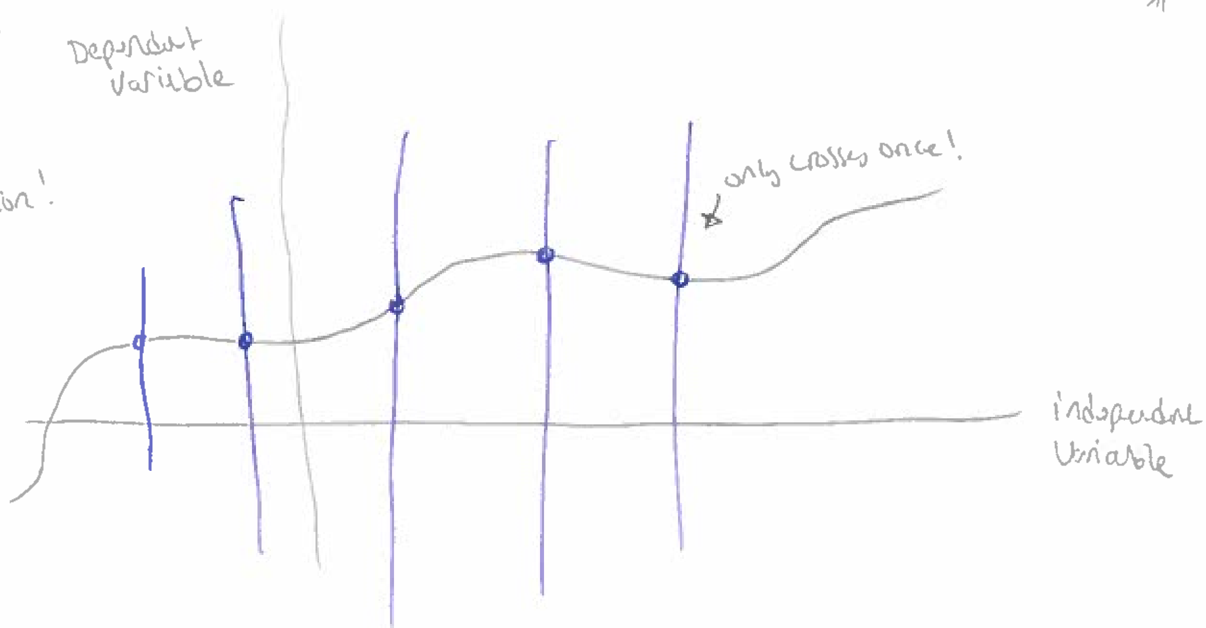
in our picture (graph) this amounts to noting that if you put a vertical line any where on the picture the graph would only cross once...

* we call this the **Vertical line Test** *

ex)

works!

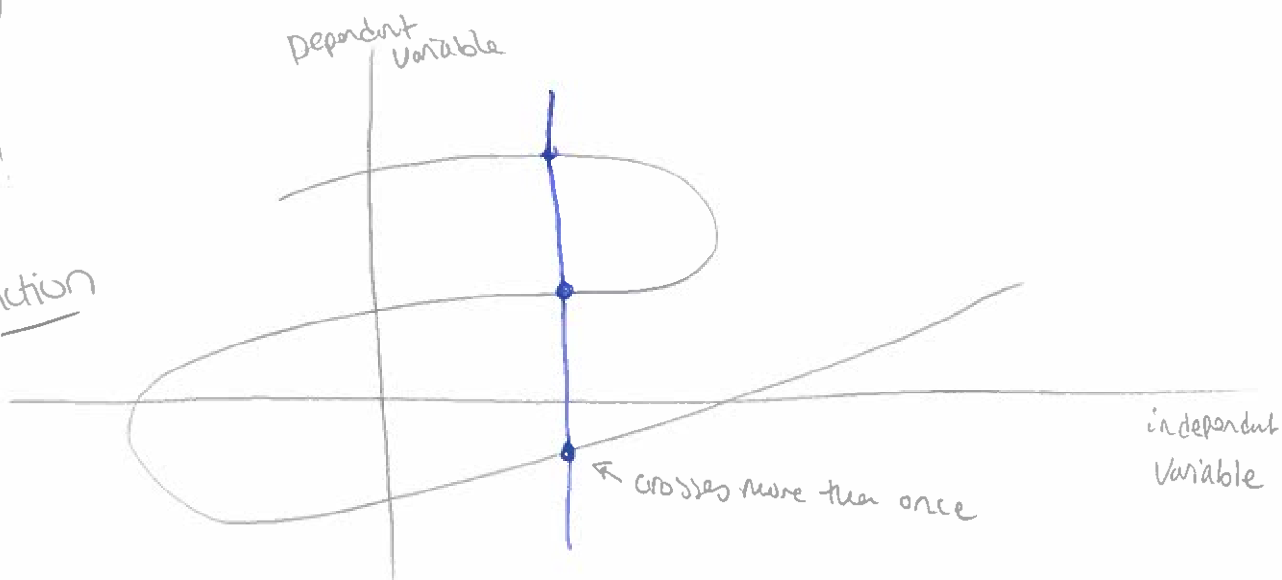
so is a function!



ex)

Doesn't work!

i.e. Not a Function

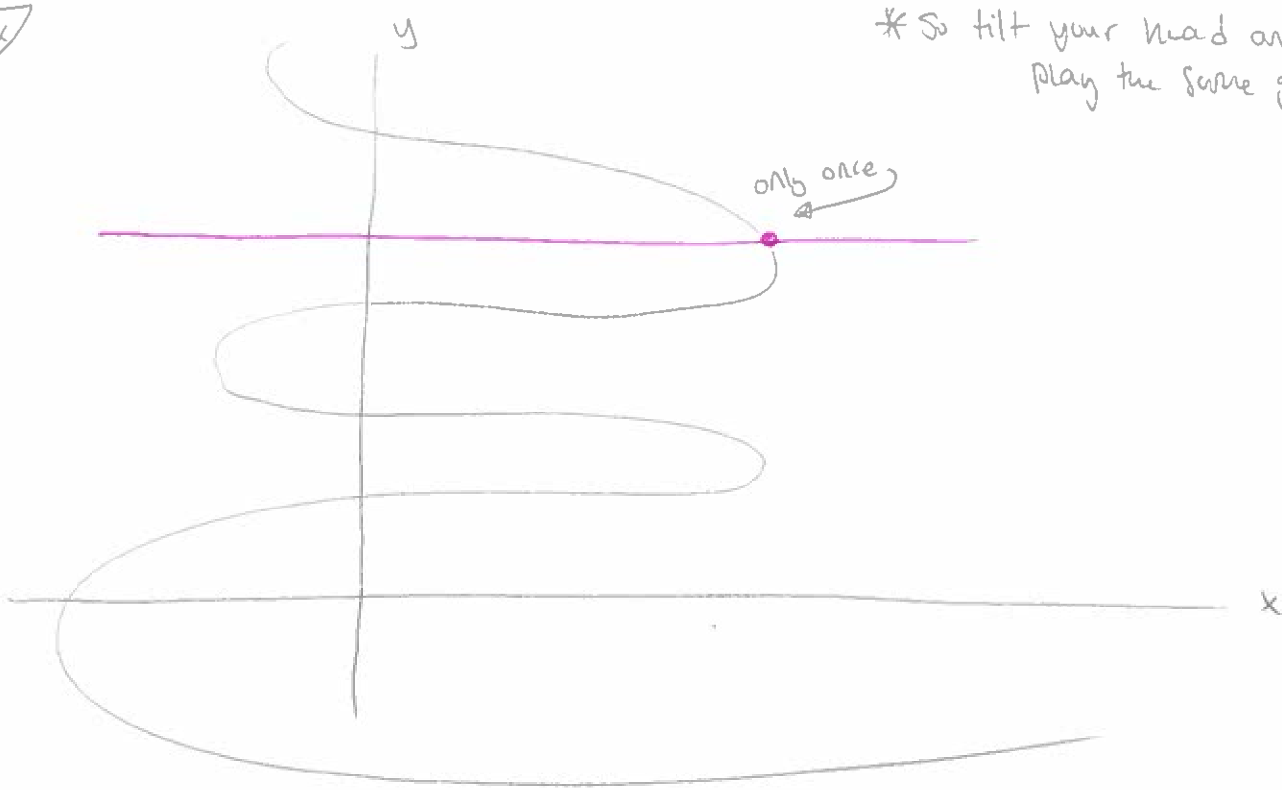


Functions

So it was a Choice where we put the dependent and the independent variable, let's look at that last example again...

(R.V.)

ex



* So tilt your head and play the same game...

So this would be a function if the independent variable was y and the dependent variable was x

recall we indicate this by saying

in english:

x depends on y

in Math:

x is a function of y

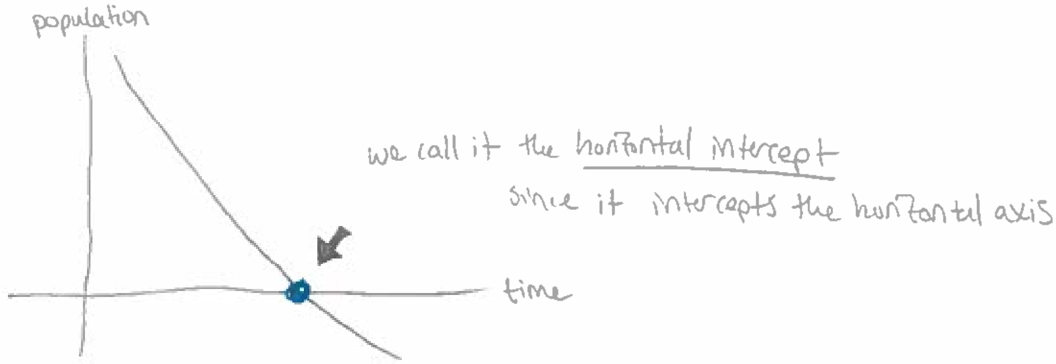
* for obvious reasons we call this the horizontal line test

Functions The picture (graph) of the function really effects the language we use to describe the function. (R.V.)

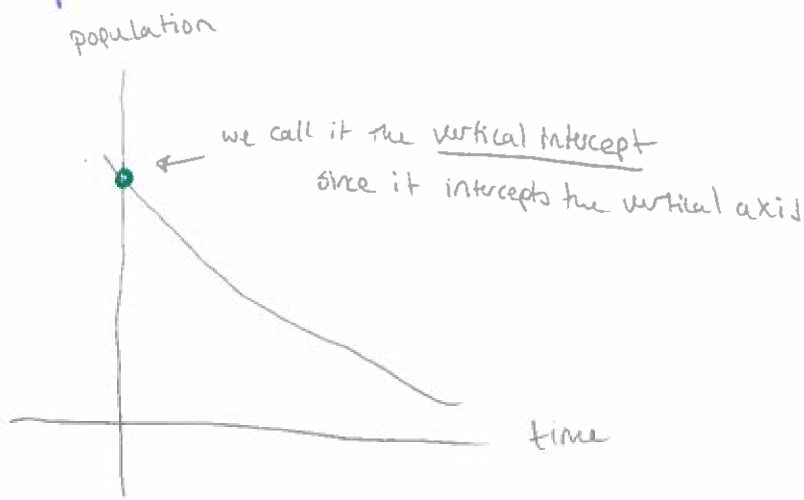
ex let's say we had a model (function) of the population dependent on only time that is

population = $P(t)$

★ Say we were curious about when (at what time, i.e. at what value of t) the population is zero, written $P(t) = 0$
the picture looks like

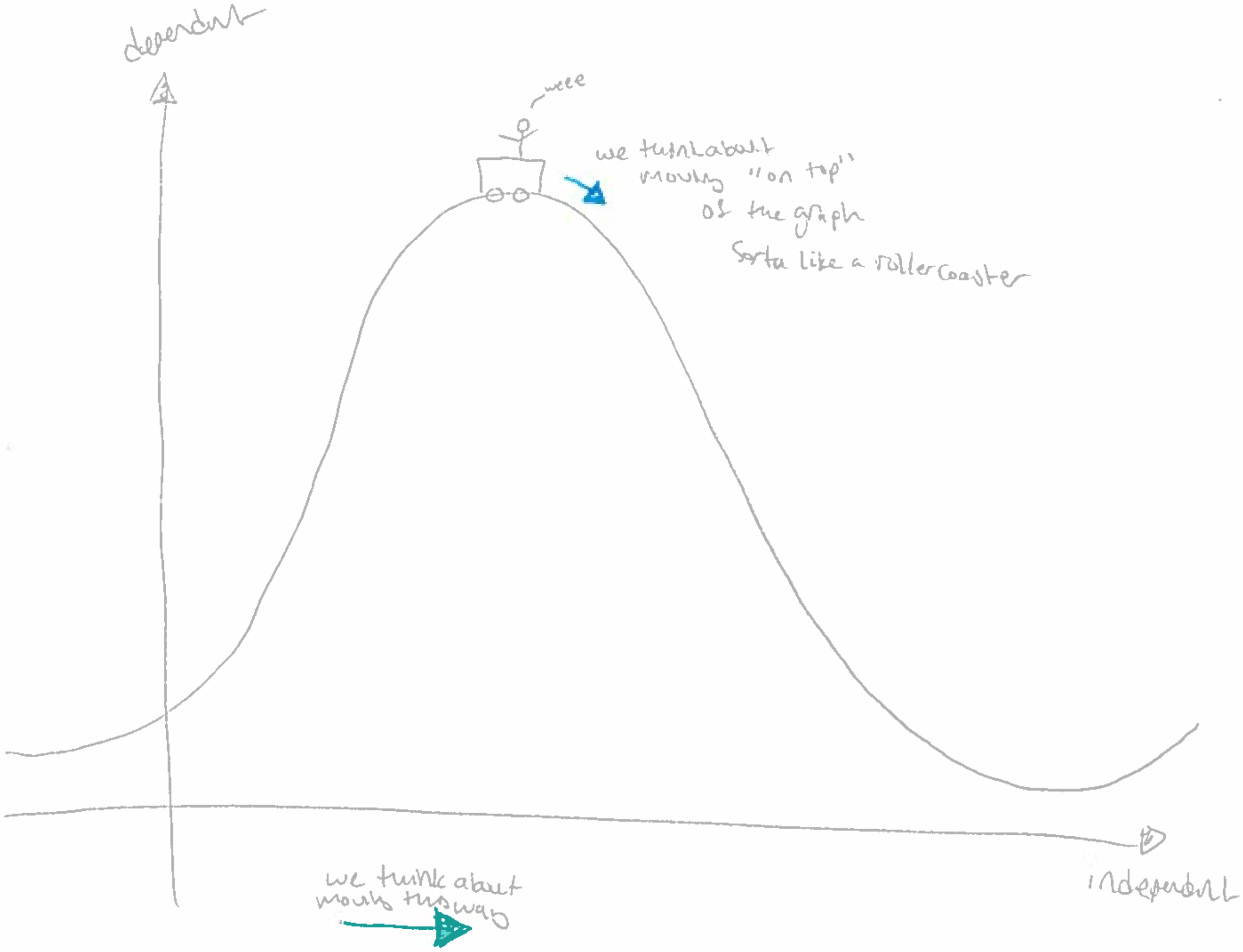


★ or say we were curious what the initial population (the population at time zero or $t=0$) written as $P(0)$ the picture looks like



⑧ Functions time was one of the first independent variables we used (I.V.)

and time has a natural "motion" so we "pretend" all independent variables have this same "motion" [i.e. times "moves forward"]



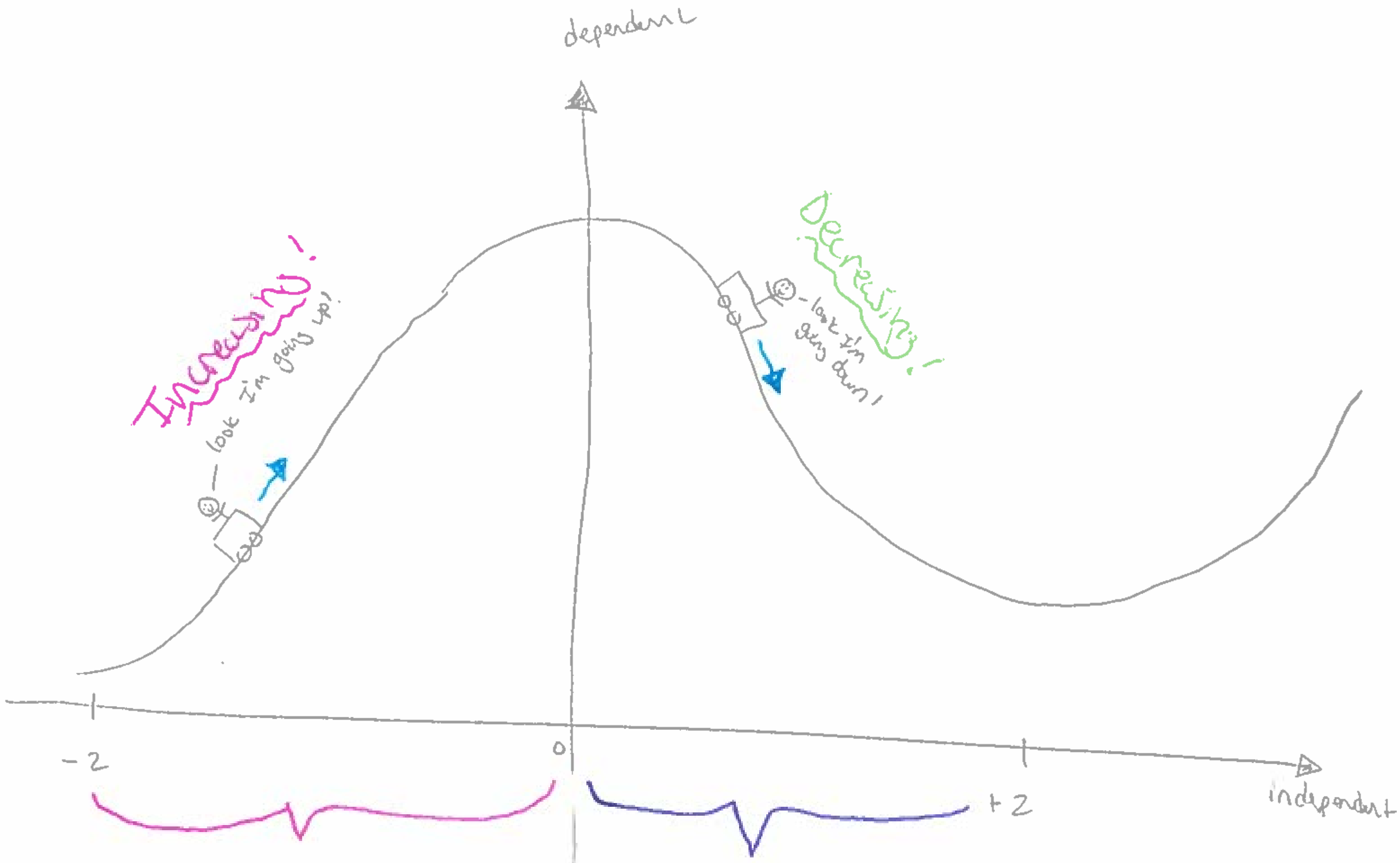
with this roller coaster "picture" in mind we can use descriptive words to describe a function like:

- increasing / decreasing
- maximum / minimum

Functions

$\boxed{ex}$ increasing / Decreasing / interval notation

The "direction of time"



The function is increasing from -2 to 0

(look we use the correct direction)

the function is decreasing from 0 to 2

interval notation:

not included!

$(-2, 0)$

start end

interval notation

$(0, 2)$