Inequalities

In this section we will solve inequalities that involve rational expressions. The process for solving rational inequalities is nearly identical to the process for solving polynomial inequalities with a few minor differences. So lets see an example of both cases.

Example 1

Solve: $\left(x+1\right)\left(x-5\right) \leq 0$

First we could try and multiply out but this will turn out not to help us, indeed if we had a polynomial we would first like to factor it before figuring out how to solve this inequality.

Since we don't know the value of x we can't divide both sides by anything that contains an x . Recall that if we multiply (or divide) both sides of an inequality by a negative number we will need to switch the direction of the inequality.

The first step is to get a zero on one side and write the other side as a single inequality. This has already been done for us here.

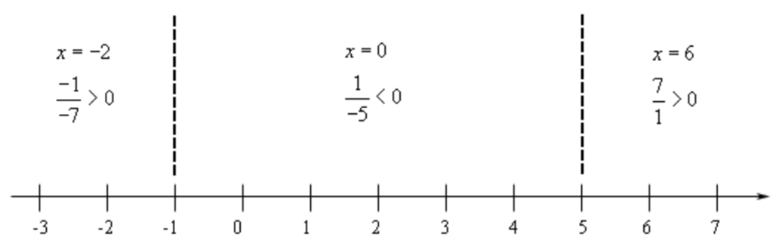
The next step is to factor as much as possible. Again, this has already been done for us in this case.

The next step is to determine where this polynomial is zero. In this case these values are.

Left Factor: $x = -1$ **Right Factor:** $x = 5$

Now, we need these numbers since, these are the only numbers where the expression may change sign. So, well build a number line using these points to define ranges out of which to pick test points just like we did with polynomial inequalities.

Here is the number line for this inequality.



So, we need regions that make the rational expression negative. That means the middle region. Also, since weve got an or equal to part in the inequality we also need to include where the inequality is zero, so this means we include $x = -1$. Notice that we will also need to avoid $x = 5$ since that gives division by zero.

The solution for this inequality is,

$$-1 \leq x \leq 5 \qquad \text{or another way of writing this...} \qquad [-1, 5]$$

Problem 1. For the following functions determine what values of x the outputs are positive.

(a) $f(x) = -x^2 + 4x + 21$ (b) $g(x) = (x-1)^2(x+2)(x+3)$ (c) $h(x) = \frac{2x+3}{x^2-4}$

Example 2

Solve: $\dfrac{x+1}{x-5} \le 0$

Before we get into solving these we need to point out that these DON'T solve in the same way that we've solved equations that contained rational expressions. With equations (i.e. equalities) the first thing that we always did was clear out the denominators by multiplying by the least common denominator. That wont work with these however.

Since we don't know the value of x we cant multiply both sides by anything that contains an x . Recall that if we multiply both sides of an inequality by a negative number we will need to switch the direction of the inequality. However, since we dont know the value of x we dont know if the denominator is positive or negative and so we wont know if we need to switch the direction of the inequality or not. In fact, to make matters worse, the denominator will be both positive and negative for values of x in the solution and so that will create real problems.

So, we need to leave the rational expression in the inequality.

Now, the basic process here is the same as with polynomial inequalities. The first step is to get a zero on one side and write the other side as a single rational inequality. This has already been done for us here.

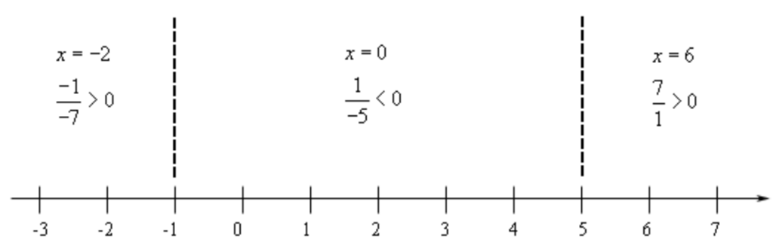
The next step is to factor the numerator and denominator as much as possible. Again, this has already been done for us in this case.

The next step is to determine where both the numerator and the denominator are zero. In this case these values are.

Numerator: $x = -1$ **Denominator:** $x = 5$

Now, we need these numbers for a couple of reasons. First, just like with polynomial inequalities these are the only numbers where the rational expression may change sign. So, well build a number line using these points to define ranges out of which to pick test points just like we did with polynomial inequalities.

There is another reason for needing the value of x that make the denominator zero however. No matter what else is going on here we do have a rational expression and that means we need to avoid division by zero and so knowing where the denominator is zero will give us the values of x to avoid for this.
Here is the number line for this inequality.



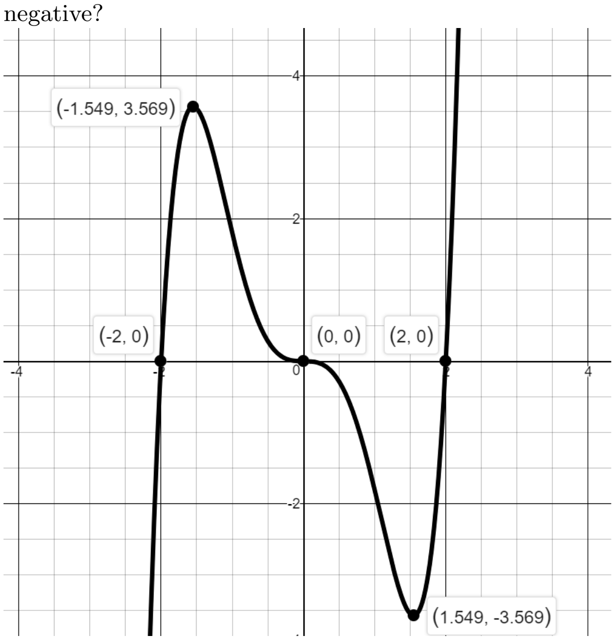
So, we need regions that make the rational expression negative. That means the middle region. Also, since weve got an or equal to part in the inequality we also need to include where the inequality is zero, so this means we include $x = -1$. Notice that we will also need to avoid $x = 5$ since that gives division by zero.

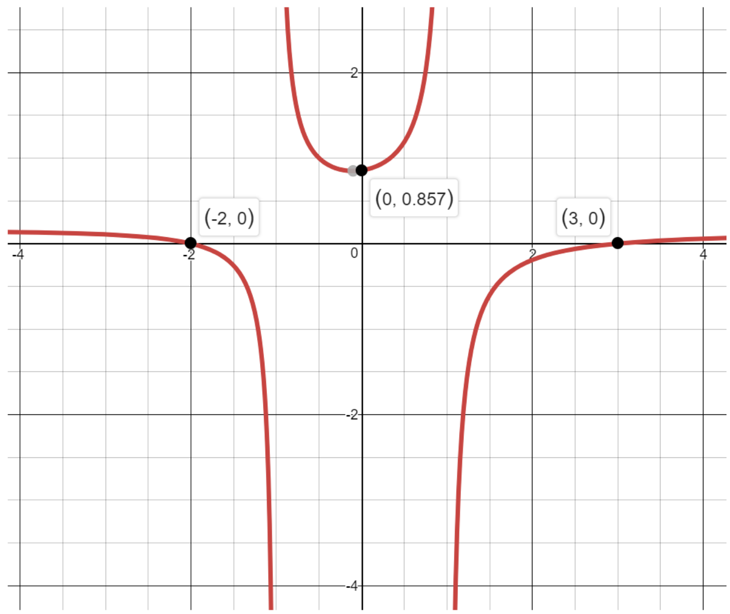
The solution for this inequality is,

$-1 \le x < 5$ or another way of writing this... $[-1, 5)$

How did these two examples differ? How are they the same??

Problem 2. For the following graphs determine for what values of x are the output positive and for what values of x are the output negative?





Problem 3. Solve the following inequalites.

(a) $x^2 + 2x + 1 > 0$

(b) $10x^2 < 3 - 13x$

(c) $\frac{3x+1}{(x-2)(x-3)} \geq 0$

(d) $\frac{2}{x+1} \leq 4$

(e) $\dfrac{4-x}{x+3} > 0$

(f) $\dfrac{2z-5}{z-7} \leq 0$